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Decoding of Concatenated Codes for Noisy Channels With Unknown Offset

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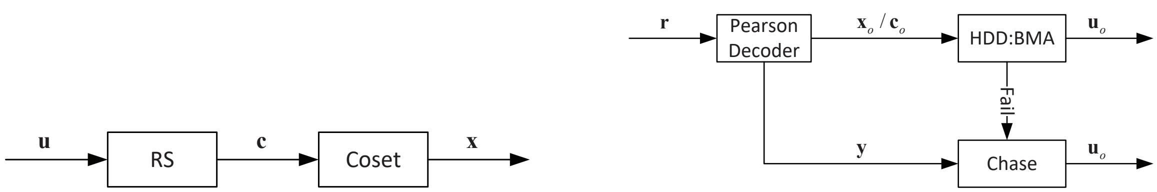
In communication and storage systems, noise and interference are not the only disturbances during the data transmission, sometimes the error performance is also seriously degraded by offset mismatch. We consider a simple channel such that the received signal is distorted by noise and offset mismatch, that is, $\mathbf{r} = \mathbf{x}+\mathbf{v}+b\mathbf{1}$, where $\mathbf{x} = (x_1, x_2, \ldots, x_n)$ is the transmitted codeword from a codebook, $\mathbf{v} = (v_1, v_2, \ldots, v_n) \in \mathbb{R}^n$ is the noise vector, where the v_i are independently normally distributed with mean 0 and standard deviation σ, b is a real number representing the channel offset, $\mathbf{1}$ is the real all-one vector $(1, \ldots, 1)$ of length n, and $\mathbf{r} \in \mathbb{R}^n$ is the received vector. Minimum modified Pearson distance (MMPD) detection has been proposed [1] as an alternative to minimum Euclidean distance (MED) detection to counter the effects of offset mismatch. A major concern, however, is the fact that the evaluation of MMPD is an exhaustive search over all candidate codewords which is infeasible for large codes. Various block codes have been proposed [2] to get good performance for channels with both noise and offset if the MMPD detection is used.



(a) Block diagram of the RS-Coset encoder (b) Block diagram of the decoding algorithm

Figure 1: Block diagram of concatenated scheme.

In this work, a concatenated coding scheme is proposed for noisy channels with unknown offset mismatch. The concatenation is between a Reed-Solomon (RS) code and a certain coset of a binary block code proposed in [2]. The two codes are chosen according to a rule that the inner code is of a short length such that an exhaustive search of MMPD is for relatively small codes. The encoder block diagram of RS-Coset codes is shown in Figure 1(a). A message vector $\mathbf{u}$ is encoded to a RS codeword $\mathbf{c}$. Then it will be converted into a binary sequence and mapped to a codeword $\mathbf{x}$ in a coset of a block code. A novel soft decoding algorithm shown in Figure 1(b) for the concatenated scheme is proposed. The MMPD detection is used to decode the inner code that guarantees immunity to channel offset mismatch. Its output will be given to a two-stage hybrid decoding algorithm for the outer RS code. The first stage of decoding carries out an algebraic hard-decision decoding (HDD) algorithm, such as the Berlekamp-Massey algorithm (BMA), to the outer RS code. If the HDD declares a successful decoding, then the algorithm outputs the decoded codeword and terminates the decoding process. Otherwise, the decoding is continued with the second stage of decoding, using the reduced test-pattern Chase algorithm. The reliability information $\mathbf{y}$ used in Chase algorithm is obtained from a subtraction between the received vector and an estimated offset, where a dynamic threshold estimation [3] is explored.

As an example, the performance of the proposed concatenation scheme that implements $(7, 3, 5)$ RS code as the outer code and the coset of binary $(6, 3, 3)$ code as the

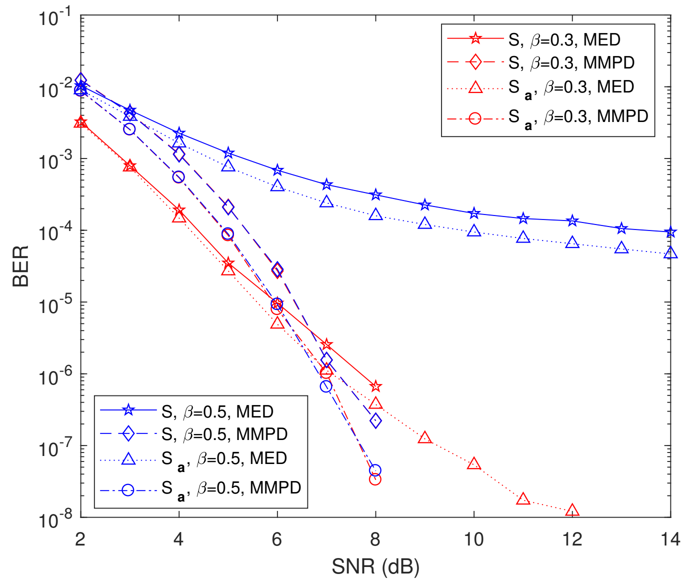
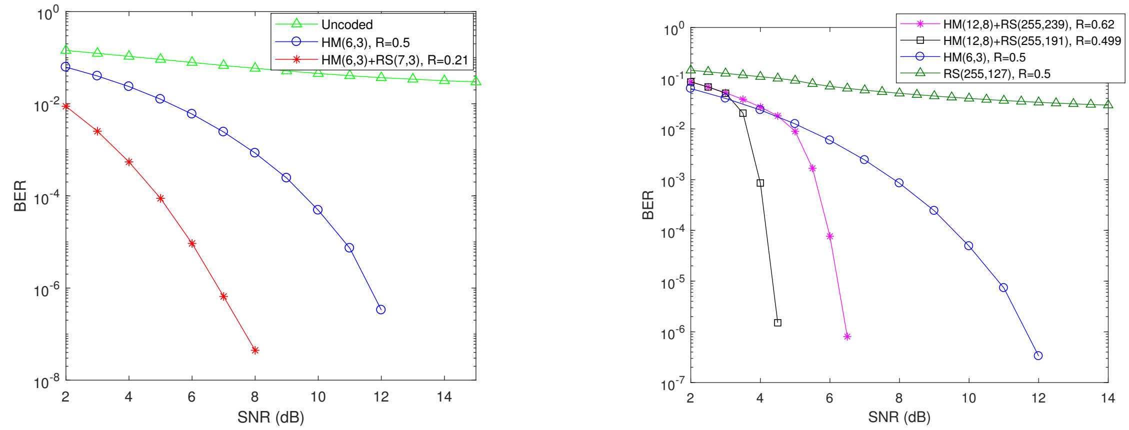


Figure 2: BER performances of (7,3,5) RS code concatenated with shortened (6,3,3) Hamming code or its coset, with the inner soft decision decoder based on two criteria – MED and MMPD – over channels with Gaussian noise and offset, where standard deviations of the offset are $\beta = 0.5$ (blue) and 0.3 (red).

inner code is evaluated. Binary $(6, 3, 3)$ code is a shortened version of $(7, 4, 3)$ Hamming code and a coset vector $\mathbf{a} = (1, 0, 0, 0, 0, 0)$ is considered. In Figure 2, we show bit error rate (BER) performances of RS-Coset codes versus signal to noise ratio (SNR (dB)) over channels with Gaussian noise and offset mismatch. We also assume that standard deviations of the offset are $\beta = 0.3$ (red curves) and $\beta = 0.5$ (blue curves). Let us first compare the performance of the inner soft-decision decoders based on two criteria, specifically, MED and MMPD. The MED detection has worse performance when the offset is larger, while the scheme using MMPD remains the same for any offset as we expected. We conclude that the MMPD detection is immune to the offset mismatch and achieves considerable performance improvements, particularly when the offset is large compared to the noise. We also observe that with the Hamming coset code as inner code instead of Hamming code itself, the simulated results of the concatenated scheme have been improved. The MMPD of the shortened Hamming code $(6, 3, 3)$ and its coset code is the same, however, the average number of neighbors with the minimum distance has decreased after introducing the coset of code. In addition, the introduction of the coset increases the MED detection's resistance of the offset mismatch. Thus, the performances of both MED and MMPD detection have improved by using the coset of block codes as proved in [2].

Figure 3(a) compares the proposed concatenated scheme with the Hamming coset code and uncoded scheme. Simulation is carried out over channels with $\beta = 0.5$. The Hamming coset code is decoded using the MMPD scheme. By referring to the channel raw BER illustrated by Curve Uncoded, the proposed concatenated scheme achieves a significant gain in BER over a wide range of SNR. Furthermore, compared with the Hamming coset code, it can be observed that at BER $= 10^{-4}$, the gain of the concatenated scheme corresponds to the decrease of the system required SNR from around 9 dB (corresponding to the HM(6,3)) to 5 dB (corresponding to the HM(6,3)+RS(7,3)). Thus, we achieve more than 4 dB SNR improvement of achieving a BER $= 10^{-4}$ with the proposed RS-Coset codes.

Our example code has a code rate of 0.21, which is lower than the non-concatenated



(a) BER performances of $(7,3,5)$ RS code concatenated with coset shortened Hamming (6,3,3), only coset Hamming (6,3,3), and uncoded case.

(b) BER performances of different coding schemes.

Figure 3: Performance evaluation over channels with Gaussian noise and offset, where the standard deviation of the offset is $\beta = 0.5$.

scheme. Here, we have other two concatenation schemes with higher code rates: coset shortened Hamming (12,8) concatenated with (255,191) RS with a code rate of 0.499; coset shortened Hamming (12,8) concatenated with (255,239) RS with a code rate of 0.62. Observe from Figure 3(b), the concatenation codes with a higher rate are shown to have worse BER performance. However, the concatenated scheme with a code rate of 0.62 still performs better than the Hamming coset code, which has a code rate of 0.5. What's more, (255,127) RS code is simulated under the same channel condition, whose code rate is 0.5. We can see that its performance is seriously distorted by the offset mismatch without the help of the inner MMPD decoder. The simulation results show the considerable coding gain compared with the non-concatenated codes with an even higher code rate over noisy channels with offset mismatch. Thus we conclude that the RS-Coset codes with the proposed decoding scheme can achieve great coding gain and also maintain the immunity to offset mismatch.

References

[1] K. A. S. Immink and J. H. Weber, "Minimum Pearson Distance Detection for Multilevel Channels with Gain and/or Offset Mismatch," IEEE Transactions on Information Theory, Vol. 60, no. 10, pp. 5966-5974, Oct. 2014.

[2] J. H. Weber, R. Bu, K. Cai and K. A. S. Immink, "Binary Block Codes for Noisy Channels with Unknown Offset," IEEE Transactions on Communications, vol. 68, no. 7, pp. 3975-3983, 2020.

[3] K. A. S. Immink, K. Cai and J. H. Weber, "Dynamic Threshold Detection Based on Pearson Distance Detection," IEEE Transactions on Communications, vol. 66, no. 7, pp. 2958-2965, Jul. 2018.