TU Delft

Delft University of Technology

Document Version
Final published version

Licence
CC BY-NC-ND

Citation (APA)
Bakker, M., & Brakenhoff, D. (2026). Timflow: Multi-Layer Analytic Element Modeling in Continuous Space and Time. *Groundwater*. https://doi.org/10.1111/gwat.70109

Important note
To cite this publication, please use the final published version (if applicable).
Please check the document version above.

Copyright
In case the licence states “Dutch Copyright Act (Article 25fa)”, this publication was made available Green Open Access via the TU Delft Institutional Repository pursuant to Dutch Copyright Act (Article 25fa, the Taverne amendment). This provision does not affect copyright ownership.
Unless copyright is transferred by contract or statute, it remains with the copyright holder.

Sharing and reuse
Other than for strictly personal use, it is not permitted to download, forward or distribute the text or part of it, without the consent of the author(s) and/or copyright holder(s), unless the work is under an open content license such as Creative Commons.

Takedown policy
Please contact us and provide details if you believe this document breaches copyrights.
We will remove access to the work immediately and investigate your claim.

This work is downloaded from Delft University of Technology.

Check for updates

Groundwater

Technology Spotlight/ Technology Editor / Michael N. Fienen

Timflow: Multi-Layer Analytic Element Modeling in Continuous Space and Time

Mark Bakker[1] and Davíd Brakenhoff[2,3]

Introduction

Grid-based groundwater modeling has become so standard over the years, that it may be easy to forget that the world doesn't consist of grid cells, time does not pass in steps, and most multi-aquifer systems are not bounded. Evaluation of the effects of smaller grid cells, smaller time steps, or alternative model boundaries are routine, but not very popular tasks of groundwater modelers. This is where the analytic element method comes in. The analytic element method is a gridless method based on the superposition of analytic elements. Analytic elements are analytic solutions and represent physical features in the aquifer such as a well (a point element) or a river segment (a line element). There is no grid, time is continuous, and there is no necessity for a closed model boundary.

Timflow is an open-source Python package for analytic element modeling of multi-aquifer flow. Pumping and observation wells, lakes, rivers, and other elements may be located anywhere as there is no grid. The head and flow may be computed with Timflow at any point and at any time. Analytic elements are added to represent physical features both inside and outside the model area up to a distance where their effect on the flow in the model area is negligible.

The basic idea of the analytic element method is that each analytic element has one or more free parameters, such as the discharge of a well or the inflow along a river segment. The free parameters are computed from a specified condition, for example, the water level in a river segment. The inflow along the river segment (the free parameter) is computed such that the head in the aquifer is equal to the water level in the river at a control point along the river segment. All free parameters of a model are commonly computed simultaneously, as the inflow along one river segment may affect the head in the aquifer at other river segments and vice versa.

Timflow can model three-dimensional steady and transient flow in multi-aquifer systems, and combines the previous TimML (steady flow) and TTim (transient flow) packages into one new package. Applications of Timflow vary from well field design and capture zone analysis to dewatering problems, aquifer test analysis, and stream flow depletion modeling. Timflow has a separate option to model flow in a vertical cross-section, which is used, for example, for the analysis of dike stability. In the following, steady flow modeling is described first, followed by transient flow modeling. Development and sustainability plans are discussed at the end of this paper.

Steady Flow Modeling

Timflow simulates steady groundwater flow in multi-layer systems consisting of aquifer layers and leaky layers (aquitards). The Dupuit approximation is adopted for flow in aquifer layers, that is, the resistance to flow in the vertical direction is neglected, which allows for the computation of three-dimensional pathlines (Strack 1984). Flow in leaky layers is approximated as vertical. The transmissivity of an aquifer layer and the resistance of a leaky layer (the inverse of the leakance) are constant within a layer, but may differ inside zones bounded by a polygon, called inhomogeneities. Unconfined flow is modeled with a constant transmissivity, which is often accurate (Sheets et al. 2015). Aquifers may be subdivided into multiple aquifer layers to facilitate the modeling

[1]Corresponding author: Faculty of Civil Engineering and Geosciences, Delft University of Technology, Delft, Netherlands; mark.bakker@tudelft.nl

[2]Faculty of Civil Engineering and Geosciences, Delft University of Technology, Delft, Netherlands

[3]Artesia B.V, Schoonhoven, Netherlands

Article impact statement: Timflow is an open-source Python package for steady and transient analytic element modeling of groundwater flow in multi-layer systems.

Received September 2026, accepted September 2026.

© 2026 The Author(s). *Groundwater* published by Wiley Periodicals LLC on behalf of National Ground Water Association.

This is an open access article under the terms of the Creative Commons Attribution-NonCommercial-NoDerivs License, which permits use and distribution in any medium, provided the original work is properly cited, the use is non-commercial and no modifications or adaptations are made.

doi: 10.1111/gwat.70109

of partially penetrating wells and streams, or horizontal wells. The flow between aquifer layers that make up one aquifer is computed with a finite-difference scheme using the vertical component of the hydraulic conductivity. The top boundary of the multi-aquifer system may be either confined (with optionally a uniform areal recharge inside inhomogeneities) or semi-confined, representing a leaky layer with a fixed head above it. Details of the steady flow formulation used in Timflow can be found in Bakker and Strack (2003).

Timflow includes point and line elements. Point elements (a point in the two-dimensional horizontal plane) are used to simulate flow to pumping wells (either extraction or injection). Wells may be screened in multiple aquifer layers, in which case the total discharge of the well is divided (by Timflow) over the aquifer layers such that the head inside the well is the same in all screened layers; resistance of the well screen (skin effect) may be taken into account. Alternatively, Timflow can compute the discharge of the well from a head condition. The head may be specified inside the well or at a target location, which can be used to answer the question "How much should this well pump such that the head at the target location is equal to X." A target head may also be specified for a string of wells, for example, for dewatering a building pit with a vacuum system, where all wells are connected by a header pipe and have the same (a priori unknown) head.

The most versatile line element in Timflow is termed a line-sink in analytic element lingo. As may be inferred from the name, a line-sink takes water out of the aquifer (or puts water in) along a line segment. The inflow along line-sinks in Timflow varies as a polynomial of specified order. Strings of line-sinks may be used to simulate a variety of features, including:

1. Narrow rivers and streams. The water level is specified at control points along the river segment including (optionally) the resistance of the leaky streambed.
2. Ditches. The total discharge of the string of ditches is specified. The resulting water level in the ditch is uniform and computed by the model.
3. Horizontal wells and collector wells. The aquifer is divided into multiple aquifer layers such that the thickness of the aquifer layer is equal to the diameter of the well at the depth of the horizontal well. The total discharge of the well is specified and the inflow is distributed along the horizontal well or arms of the collector well such that the head inside the well is the same everywhere.
4. Boundaries of inhomogeneities. Zones with different aquifer or leaky layer properties are bounded by line elements so that the head and the normal component of flow are continuous across the boundary of the inhomogeneity. The top boundary of the aquifer system may also differ inside an inhomogeneity, for example to simulate a lake with a leaky lake bottom.

Leaky or impermeable walls may be modeled with another type of line element called a line-doublet. Such an element can be used to create a jump in the head across a line segment. The jump in head is computed by enforcing the appropriate boundary condition (e.g., no flow for impermeable walls) at control points along the line element. Strings of leaky or impermeable wall elements may be screened in one or more layers. Applications include rock outcrops or the modeling of sheet piles along building pits.

Timflow can compute three-dimensional pathlines by numerically integrating the analytic velocity field, for example to simulate the advective transport of contaminant plumes or to compute the time of travel from surface water bodies to pumping wells. Capture zones may be simulated by starting a number of pathlines around a well screen and tracing them against the flow.

Steady Flow Example

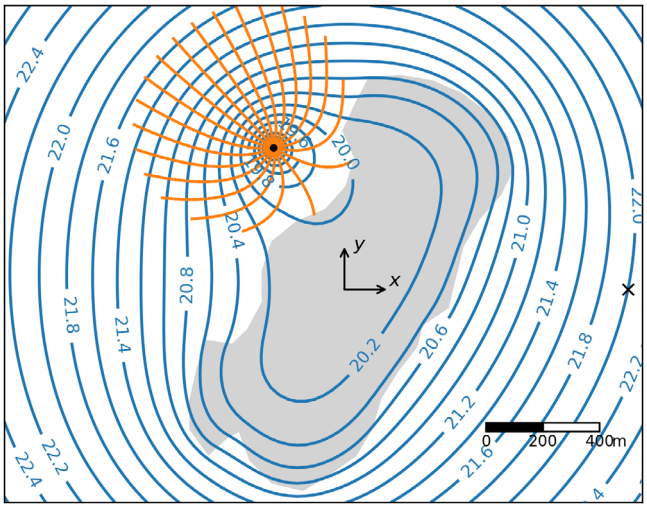
A steady flow example is presented for a pumping well near a lake in an unconfined aquifer with a bottom elevation set to 0 m. The hydraulic conductivity of the aquifer is 10 m/d and the thickness is 20 m except for under the lake where the thickness is 19 m and the flow is semi-confined. The lake bottom consists of a 1-m thick layer with a vertical hydraulic conductivity of 1 cm/d, so that the resistance is 100 d (leakance of $0.01\,d^{-1}$). The lake level is 20 m and the head some distance to the right of the lake is 22 m (at the × in Figure 1). The water level in the well is specified as 16 m, 4 m below the lake level. Timflow computes the discharge of the well as $816\,m^3/d$. A contour plot is created and a 10-year capture zone is computed with 20 pathlines (Figure 1). Part of the water pumped by the well comes from the lake. Lake water infiltrates into the aquifer inside the 20 m contour line.

Transient Flow Modeling

Transient groundwater flow is simulated in the same multi-aquifer system as for steady flow. For transient flow, elastic storage (specific storage) must be specified for all aquifer layers and leaky layers except for the top aquifer layer which may optionally have phreatic storage (specific yield). Aquifer properties are constant in space and time; transient inhomogeneities are not (yet) implemented. Storage in leaky layers is taken into account without the need to subdivide the leaky layer into multiple model layers. The Laplace-transform analytic element method is applied (Kuhlman and Neuman 2009). Under the hood, an analytic element solution is computed in the Laplace domain and the time domain solution is obtained numerically through application of the algorithm of De Hoog et al. (1982). Once a Timflow solution is obtained, the head can be computed at any location in the aquifer and at any time. Details of the transient flow formulation used in Timflow are described in Bakker (2013).

A transient Timflow model is a difference model: the transient response is simulated starting with no transient response at time zero (i.e., the initial condition is zero head difference everywhere). This means that when a well

17456584, 0, Downloaded from https://ngwa.onlinelibrary.wiley.com/doi/10.1111/gwat.70109 by Technical University Delft, Wiley Online Library on [28/10/2026]. See the Terms and Conditions (https://onlinelibrary.wiley.com/terms-and-conditions) on Wiley Online Library for rules of use; OA articles are governed by the applicable Creative Commons License




```python
import timflow.steady as tfs
# create model and store in variable ml
ml = tfs.ModelMaq(kaq=10, z=[20, 0])
# add lake to model ml
# xylake is array with coordinates of perimeter
lake = tfs.PolygonInhomMaq(model=ml, xy=xylake, kaq=10,
                           z=[20, 19, 0], c=100,
                           topboundary='semi', hstar=20)
# head fixed at reference point (x in figure)
rf = tfs.Constant(model=ml, xr=1000, yr=0, hr=22)
# well with specified head -> Q is computed
w = tfs.HeadWell(model=ml, xw=-250, yw=500,
                 hw=16, rw=0.3)
ml.solve() # solve for all free parameters
# plot contours
ax = ml.plots.contour(win=[-1200, 1200, -800, 1000],
                      levels=np.arange(16, 24, 0.2))
# compute and plot 10-year capture zone for well w
ml.plots.plot_capture_zone(well=w, nt=20,
                           tmax=10 * 365, ax=ax)
```

Figure 1. Steady flow example for a pumping well near a lake. Head contours and 10-year capture zone (left) and the Timflow script that creates the model and figure (right).

starts pumping near a river with a constant stage, the head difference along the river is specified as zero (or "fixed"), as the stage does not change during pumping. Transient Timflow includes the same types of elements as steady Timflow, but boundary conditions, such as the discharge of a pumping well, must be specified by step-wise time series. Step changes can occur at arbitrary times. The total head may be obtained as the sum of a steady and a transient model.

Timflow includes calibration functionality to estimate aquifer parameters from pumping tests and slug tests by applying a standard weighted least-squares approach. Timflow can analyze tests in very general settings, including variable pumping rates, wellbore storage, an arbitrary number of observation wells, rainfall, and curved rivers with a variable stage. Timflow results for a number of aquifer tests are available on the Timflow documentation site, including comparisons to results of alternative (commercial) software. Parameter estimation in Timflow is not restricted to aquifer tests but can be applied to any Timflow model.

Transient Flow Example

The transient flow example concerns the computation of stream flow depletion in a system of two aquifers separated by a leaky layer with resistance c. The top aquifer is unconfined and is modeled with a constant transmissivity and phreatic storage. All aquifer parameters are given in the Timflow script in Figure 2. The stream has a constant level and is in full contact with the top aquifer. A pumping well starts pumping from the bottom aquifer at 500 m^3/d at time zero, and is reduced to 300 m^3/d after 100 d. Head contours in both aquifers after 100 d of pumping are shown on the left side of Figure 2. The (specified) discharge of the well, and the (computed) head in the well and the stream flow depletion (the total discharge coming from the river) are shown as a function of time in the lower-right-side graph.

Limitations

Not surprisingly, Timflow also has limitations. Timflow is written in Python using Numba to speed up computations. Run times are short but increase linearly with the number of elements, although run times also reduce almost linearly with the number of computer cores. Models with hundreds of inhomogeneities to represent spatially variable aquifer properties will probably take longer to run than most users have the patience for.

Line elements need appropriate discretization. Line elements are analytic solutions for a certain distribution of the inflow (line-sinks) or head jump (line-doublets). These elements are used to approximate a different boundary condition, for example a constant head or a zero normal component of flow. These conditions are met exactly by Timflow at control points along the line element, but approximately between them. The condition may be met more accurately by using shorter line elements and/or line elements with multiple control points (higher order elements).

Timflow is based on a mathematical formulation for linear systems. The simulation of nonlinear systems is not possible. For example, a saturated thickness that is a function of the head results in a nonlinear system and cannot be simulated. Likewise, ephemeral streams cannot be simulated (although there are some exciting development plans to include them approximately in a future version).

Future Developments and Sustainability

Timflow is open-source software hosted on GitHub (see the Data Availability section for links) which includes a Discussion section where users can submit and respond to Timflow discussions. The Timflow GitHub site also includes an Issues section where possible bugs can be reported. Timflow combines more than

17456584, 0, Downloaded from https://ngwa.onlinelibrary.wiley.com/doi/10.1111/gwat.70109 by Technical University Delft, Wiley Online Library on [28/10/2026]. See the Terms and Conditions (https://onlinelibrary.wiley.com/terms-and-conditions) on Wiley Online Library for rules of use; OA articles are governed by the applicable Creative Commons License

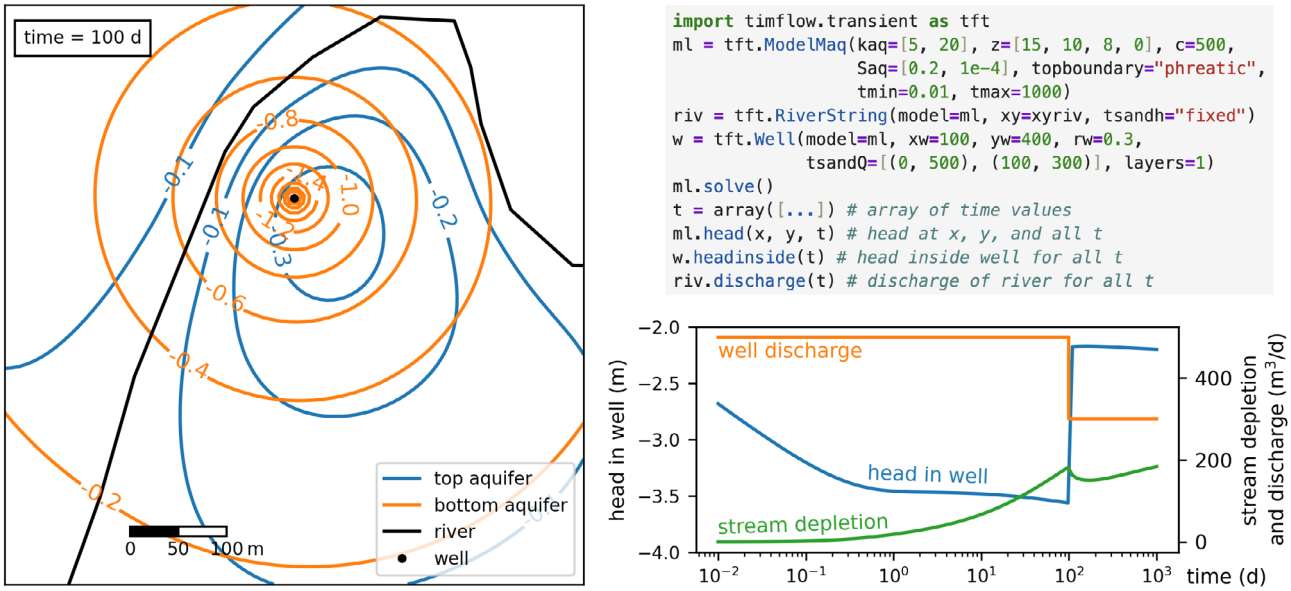


Figure 2. Transient example of stream flow depletion. Contours of head change in both aquifers after 100 d of pumping (left), Timflow script (upper right), and discharge, head in well, and stream flow depletion vs. time (lower right).

15 years of multi-layer analytic element developments. Addition of new elements and features is a function of available time, funds, and inspiration. Both the Discussion and Issues sections are good places to read about the current developments that the Timflow team is working on.

Timflow models consist of relatively short (and readable) Python scripts, as demonstrated in the examples. Geographical information, such as the location of wells, lakes, and rivers, may be entered using Python or, for GUI-inclined users, with a dedicated QGIS plug-in developed separately (see https://deltares.github.io/QGIS-Tim/).

Timflow is free software, but that does not mean that it does not cost any time and effort to maintain and develop the package. There is a small but strong open source community supporting Timflow, but new developers are always welcome. Timflow users are encouraged to contribute to the open-source community by, for example, engaging in discussions, helping with open issues, or improving the documentation.

Acknowledgments

This work was supported by TKI Delta Technology. The authors thank all participants for their contributions.

Data Availability Statement

Timflow is released under the MIT license and is available from https://github.com/timflow-org/timflow. Extensive documentation in the form of Jupyter Notebooks is available from https://timflow.readthedocs.io. The Timflow version at the time of writing this paper is version 0.5.0.

References

Bakker, M. 2013. Analytic modeling of transient multi-layer flow. In *Advances in Hydrogeology*, ed. P. Mishra, and K. Kuhlman, 95–114. Heidelberg: Springer.

Bakker, M., and O.D.L. Strack. 2003. Analytic elements for multiaquifer flow. *Journal of Hydrology* 271, no. 1–4: 119–129.

De Hoog, F.R., J.H. Knight, and A.N. Stokes. 1982. An improved method for numerical inversion of Laplace transforms. *SIAM Journal on Scientific and Statistical Computing* 3, no. 3: 357–366.

Kuhlman, K.L., and S.P. Neuman. 2009. Laplace-transform analytic-element method for transient porous-media flow. *Journal of Engineering Mathematics* 64, no. 2: 113–130.

Sheets, R.A., M.C. Hill, H.M. Haitjema, A.M. Provost, and J.P. Masterson. 2015. Simulation of water-table aquifers using specified saturated thickness. *Groundwater* 53, no. 1: 151–157.

Strack, O.D.L. 1984. Three-dimensional streamlines in Dupuit-Forchheimer models. *Water Resources Research* 20, no. 7: 812–822.

17456584, 0, Downloaded from https://ngwa.onlinelibrary.wiley.com/doi/10.1111/gwat.70109 by Technical University Delft, Wiley Online Library on [28/10/2026]. See the Terms and Conditions (https://onlinelibrary.wiley.com/terms-and-conditions) on Wiley Online Library for rules of use; OA articles are governed by the applicable Creative Commons License