MSc Thesis

Growth of metropolitan public transport networks.

A.A.J. Vermeulen

TUDelft

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Growth of metropolitan public transport networks.

by

A.A.J. Vermeulen

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Preface

This thesis is the final product of my graduation research project for the master's program Transport & Planning at the Delft University of Technology. It concludes a research project that I have been working on from October 2017 until June 2018. It also concludes my life as a student in Delft that started in September 2008 with the bachelor's program of Civil Engineering.

My interest in network theory and public transport started with a course led by Oded Cats. When I had do decide on a research topic one of the topics involved in my choice was on network theory and public transport. Later adjusted upon finding research by Saeid Saidi and with the help of Oded providing several more interesting papers to start a research. The subject of this research is rather abstract and theoretical, something I did not expect to do at first, and provided a welcome challenge and change of scenery to a more theoretical field of work.

I would like to thank a number of people that have supported me in achieving this result. First of all I would like to express my gratitude to my daily supervisor, Oded Cats. Oded, thank you for introducing me to network theory during the course and to several relevant topics within this field. Your feedback was always quick and insightful and the energy during meetings was positive and really helped me to continue. Whenever I was in need of some new angle you always provided one in the form of some research paper. This really helped the research, so again thank you for all the time invested.

I would like to thank Hans van Lint for presiding the graduation committee. Your feedback during the committee meetings was constructive and to the point. During the search for a suitable research topic we have discussed some options for a different topic, but I am very happy that you have been willing to join my committee as chair. The feedback helped me guide to clarify the research objective and improve the overall quality of this report.

Also a word of thanks of gratitude to Martijn Warnier as well. Your feedback during the committee meetings was positive, helpful and provided for some new insights that might have otherwise been neglected.

A.A.J. Vermeulen
Delft, February 2018

Summary

Research Objective

Goal of this master's Thesis is to determine the effects various demand distributions and operational cost functions have on the evolution and shape of a monocentric metropolitan transport network evaluated using network indicators. In order to do this an investment model network analysis framework will be developed that systematically evaluates the impact the of these input functions have on the network topology, evolution and growth of a metropolitan transport network. Knowledge in this area is currently lacking in literature and will provide insight in the relationships between demand, costs and the network evolution of monocentric metropolitan networks.

The main research question is formulated as:
"What effects do various demand distributions and operational cost functions have on the evolution and shape of a monocentric metropolitan transport network when evaluating them using network indicators."

Relevance

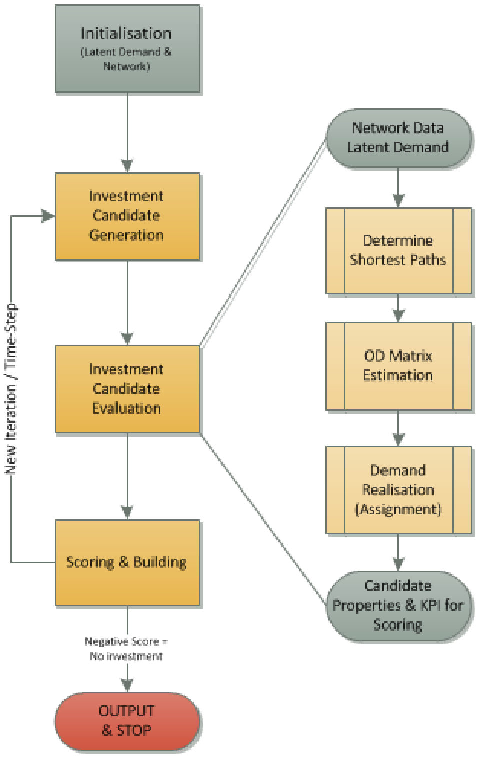
This study finds its relevance in theoretical analysis and its practical application and insights. From an theoretical perspective it is relevant to have an insight on the influences demand and cost structures have on the topological evolution of a growing transport network. This study has an explorative role in order to provide insight in these relationships given certain conditions and assumptions.

The network effects of ring-radial networks has not been the subject of many studies. Evolution or growth of transport networks is usually focused around either grid structured networks (city level) or randomly scattered networks (large world networks), leaving a clear gap in the intermediate (metropolitan region) level especially for public transport networks. Recently application of some basic theory has been conducted in the PHD thesis by Saidi (2016) and following papers with other contributors: Saidi, Ji, Cheng, Guan, Jiang, Kattan, Du, and Wirasinghe (2016a); Saidi, Wirasinghe, and Kattan (2016b); Saidi, Wirasinghe, Kattan, and Esmaeilnejad (2017), showing that this is an active field of study.

Furthermore the evolution of topological indicators in emerging/growing public transport networks has not been studied much. This research provides a framework for monocentric metropolitan public transport infrastructure growth models. A detailed chapter will relate this study to what has been done before and elaborates the scientific context of this study a little bit more.

Simulation Model

A conceptual model has been built for the simulation of the impact of the chosen population distributions and transport technologies on the topological output indicators. The investments in transport networks are based on Cost Benefit Analysis, a common tool for transport investment evaluation. The model evaluates all possible investment options and assigns a score based on the CBA. From these investments the most beneficial candidate is selected and realised within the network. This procedure is repeated until no investment with sufficient benefits exists. The flow of the simulation model is presented in the figure below:



Experiment Setup

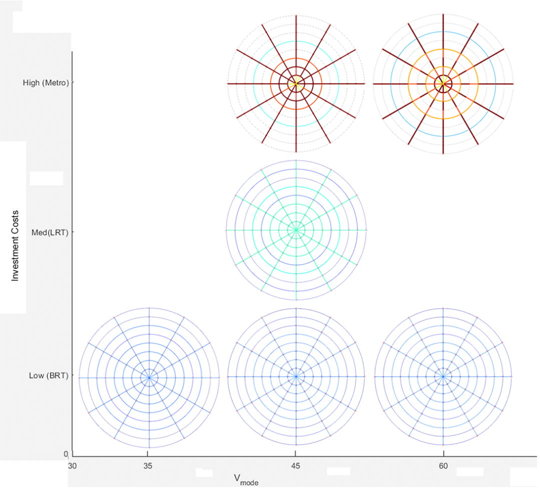
In order to conduct our research a few scenarios have been generated for experiments, a total of 9 base scenarios have been developed to investigate relationships. For each of these experiments a single variable is changed in respect to the other scenarios. These base scenarios are presented in the table below. Additionally some sensitivity analysis scenarios have been developed where the impact of costs and operation speeds is investigated in more detail, by relaxing the mode characteristics and replacing them with values found in another mode.

Scenario Name	Demand Distribution Type	Transit Technology
BU - Uniform - BRT	Uniform	Bus Rapid Transit
BL - Linear Decay - BRT	Linear Decay	Bus Rapid Transit
BE - Exponential Decay - BRT	Exponential Decay	Bus Rapid Transit
LU - Uniform - LRT	Uniform	Light Rail Transit
LL - Linear Decay - LRT	Linear Decay	Light Rail Transit
LE - Exponential Decay - LRT	Exponential Decay	Light Rail Transit
MU - Uniform - Metro	Uniform	Rapid Transit
ML - Linear Decay - Metro	Linear Decay	Rapid Transit
ME - Exponential Decay - Metro	Exponential Decay	Rapid Transit

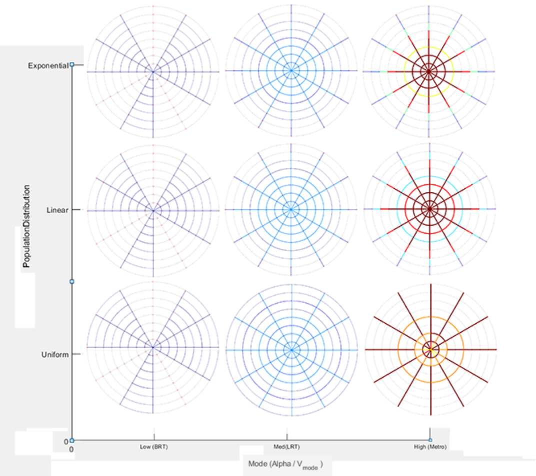
Conclusions

Population Distribution effects

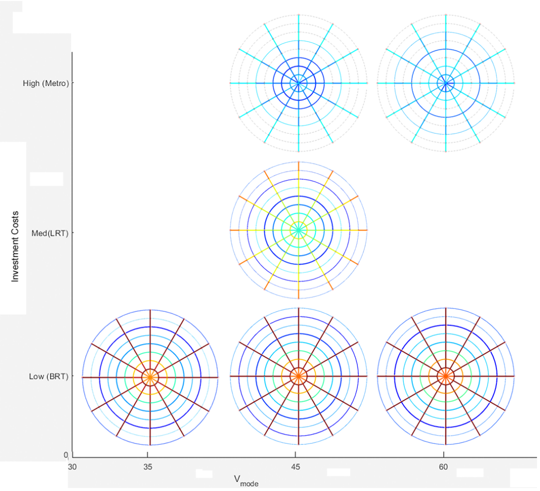
Results from the simulation model support the existence of a relationship between the population distribution and the final topological evolution of a network. Although network metrics do not account for capacity and thus do not really show these effects as clearly as expected, scenarios affected by a decaying population show limited connections in the outer periphery and a limitation of capacity for links that are constructed in these peripheral areas. This holds for all scenarios regardless of the investment cost set by transport mode parameters, meaning this effect is related to the population distribution. When looking at the key performance indicators one can see that scenarios with a linear decaying population distribution has a relatively high Average Betweenness Centrality and a detour factor that is on average higher than the uniform case. The total travel time is average placed right between the relatively high total travel time of uniform distributions and lower total travel time for exponential distributions. This might seem counter-intuitive but as more of the population is concentrated in the inner centre the relative distances between nodes is shorter and the travel time in relation to the trip distribution by the gravity model is lower, a population distribution driven effect.



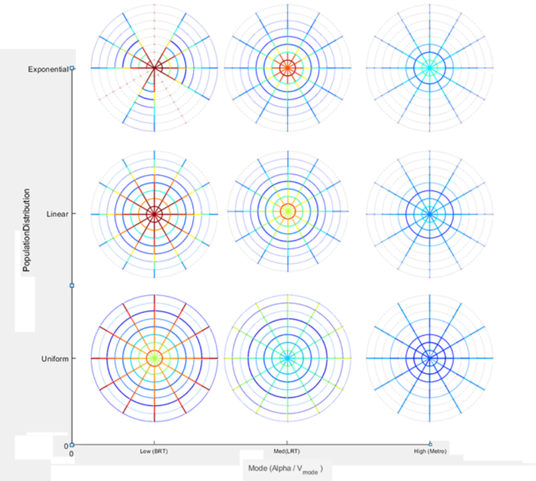
(a) Ridership for various scenarios (Cost/Speed)



(b) Ridership for various scenarios (Mode/Population)



(c) Link Loads for various scenarios (Cost/Speed)



(d) Link Loads for various scenarios (Mode/Population

Modality effects

When looking into the effects that public transport modes have on their respective network evolution and metrics of the evolution, the simulation model results clearly show the influences that mode parameters have on the final results given some of the key performance indicators. The mode affects the network evolution in two distinct ways. First of all the mode speed has an influence on the potential travel time a link has, and thus affects the travellers mode choice model. Higher speeds are beneficial up to a critical value, after that the whole potential change in modality has been realised. The exact figure for this switching point is determined by the logit choice model and the selected alternative mode characteristics, and is estimated to be within 10-20 kph difference range given the results of BRT, LRT and Metro from this model. This mechanism means that higher speeds (LRT/Metro) will likely have more of a complete network due to the potential benefits for users being higher, affecting the Cost-Benefit Analysis positively resulting in a higher ratio, thus a larger chance a link is invested in. The other investment mechanism modes affect is the costs of construction/adding capacity. These costs are a modal parameter and their effect is on the cost side of the Cost Benefit Analysis. The higher the costs per kilometre the higher the total investment costs and the more unlikely the link construction becomes. This effect is strengthened by the polar grid as in these grids the outer edges cover a relatively large distance between two equal-radii nodes. Given similar demand levels and trip likelihood, the higher costs would imply a lower CBA ratio, meaning the likelihood of construction is decreased. Radials are uniformly defined for our experiments, thus the likeliness of a ring is decreased whereas the likeliness of a radial is relatively constant assuming the trip demand is equal.

Respective evolution for scenarios can be seen in the figures below, showing the network states for fixed time moments.

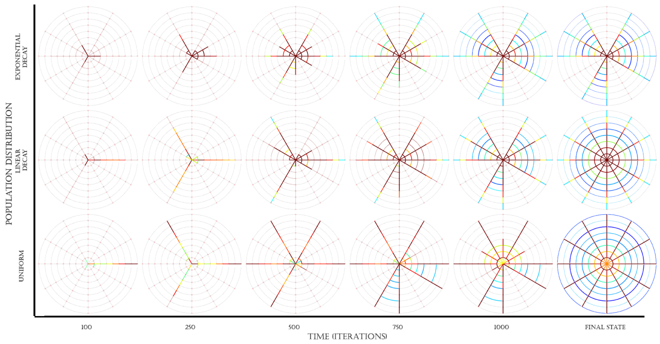


Figure 2: BRT - Network State at given Time moments

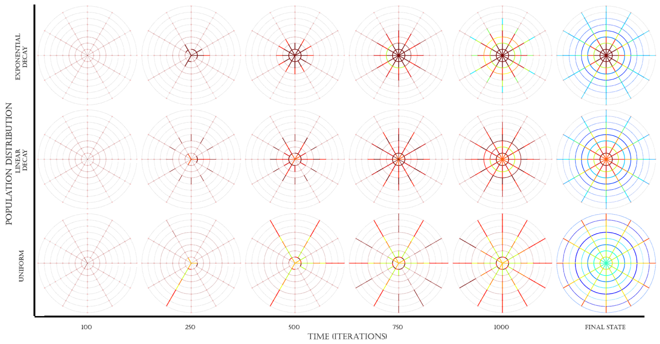


Figure 3: LRT - Network State at given Time moments

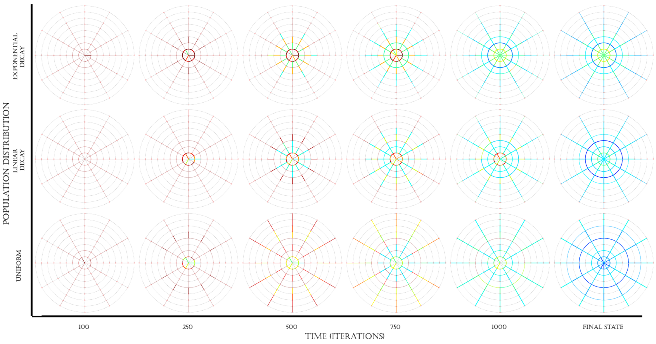


Figure 4: Metro - Network State at given Time moments

Usable Indicators for Network Evolution

Within this research various network metrics have been discussed. In order to describe the evolution of a network metrics have to be monitored over time and the resulting relative changes indicate a trend in the metrics and can help identifying specific growth mechanisms or investment choices. The final state metrics are not interesting for the evolutionary path, but can be useful when combined with intermediate time-step values. There is no single network or performance indicator that can describe the process or network state at a single time moment, however the combination of various metrics provides insight in the underlying mechanisms of network evolution within a ring-radial grid. The magnitude of some selected KPI are directly related to network grid definitions, however it is expected that the investment choices drive all of the selected KPI. Some of the indicators have overlapped thus do not really add additional information. An example of this are the β and γ indicators that both increase for same link-addition processes. Partially related to the planar grid-setup where the connections are limited and thus these metrics provide similar results.

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Introduction

1.1. Background

Extensive work exists on network planning and growth of networks; however, few studies exist on the planning and design of ring-radial rail transit systems. With more ring transit lines being planned and built in Asia, Europe and the America's, a detailed study on growth parameters for ring transit lines is required. With the high level of attention being paid to ring transit lines, the main questions that arise are the following: How do transport networks in metropolitan areas evolve over time and how can we effectively model this growth as function of demand and cost functions. In order to capture this behaviour the focus will be on Ring-Radial networks, and more specific it means investigating how adding a ring line (or a partial ring line) can be compared to adding a radial line and what role the demand and cost functions have within the growth of a network. What are the contributing factors (e.g. OD demand patterns, operator costs) that affect the topology.

The focus of this research will be the growth of monocentric urban public transport networks. More specifically idealised radio-centric networks, a theoretical simplification of Metropolitan Urban Public Transport Networks that might exist in real life. Modelling these idealised radio-centric networks can be done using a polar grid. A benefit of this is that the network shape of the model can be directly related to a radio-centric city and calculations can be simplified based on polar coordinates. The idea of using a polar grid for a network is not new, and has been used as early as in the 1960's where a polar routeing introduced by Smeed (1963) and Haight (1964) have been used to describe motor vehicle commuting patterns in cities. The evolution of a transport network on this specific type of networks have not been the topic of research as of yet. Using a polar coordinate system and polar grid do limit the options for growth or connectivity within the network. Unlike real world networks the grid is limited in its connections and not all connections can and will be considered in order to use simplify calculations. Resulting networks can be sub-optimal and can only be utilised for theoretical understanding of the topological evolution of growing public transport networks in monocentric metropolitan areas.

1.2. Research Objectives

Goal of this master's Thesis is to determine the effects various demand distributions and operational cost functions have on the evolution and shape of a monocentric metropolitan transport network evaluated using network indicators. In order to do this an investment model network analysis framework will be developed that systematically evaluates the impact the of these input functions have on the network topology, evolution and growth of a metropolitan transport network. Knowledge in this area is currently lacking in literature and will provide insight in the relationships between demand, costs and the network evolution of monocentric metropolitan networks.

1.2.1. Research Questions

- What effects do various demand distributions and operational cost functions have on the evolution and shape of a monocentric metropolitan transport network when evaluating them using network indicators.
- How do various demand distributions impact the growth of monocentric metropolitan transport networks?
- What influence do operational cost functions have on the growth of monocentric metropolitan transport networks?
- What indicators can be used to describe urban public transport network evolution?

1.2.2. Relevance of this study

This study finds its relevance in theoretical analysis and its practical application and insights. From an theoretical perspective it is relevant to have an insight on the influences demand and cost structures have on the topological evolution of a growing transport network. This study has an explorative role in order to provide insight in these relationships given certain conditions and assumptions.

As mentioned in the introduction the network effects of ring-radial networks has not been the subject of many studies. Evolution or growth of transport networks is usually focused around either grid structured networks (city level) or randomly scattered networks (large world networks), leaving a clear gap in the intermediate (metropolitan region) level especially for public transport networks. Recently application of some basic theory has been conducted in the PHD thesis by Saidi (2016) and following papers with other contributors: Saidi, Ji, Cheng, Guan, Jiang, Kattan, Du, and Wirasinghe (2016a); Saidi, Wirasinghe, and Kattan (2016b); Saidi, Wirasinghe, Kattan, and Esmaeilnejad (2017), showing that this is an active field of study.

Furthermore the evolution of topological indicators in emerging/growing public transport networks has not been studied much. This research provides a framework for monocentric metropolitan public transport infrastructure growth models. A detailed chapter will relate this study to what has been done before and elaborates the scientific context of this study a little bit more.

1.3. Knowledge Gap & Contributions of this research

As mentioned earlier the previous works still leave gaps with respect to certain areas of this study.

Saeid Saidi (2016) developed a universal model for the long-term planning and modelling of complex networks with partial ring and radial lines. This model is one of the few rind-radial network models in existence that takes various cost effects into account as the model computes generalised transit passenger cost for a given network. This means the model includes various travel-cost factors such as in-vehicle time, access and egress from stations, waiting time for passengers and transfer penalty (all from the final user point of view). This model however focuses on static evaluation of this parameter and although Saidi claims this model can be used to evaluate network investment scenarios it only works with limited scenarios. The model requires an initial network and can only be used to compare scenarios on the generalised passenger costs. It does not output the network topological indicator evolution.

The focus of the research from Lems (2017) is investment policy and the decisions made by politicians to grow a network. Obviously some cost and demand factors where compared but the focus of this study is different than that of our own. The network described is not a ring-radial structure but a generic spatial network later applied to European continent. This therefore falls in another scale and investments for larger regions cannot simply be transferred to the metropolitan level. Within his research Lems models cost effects in a simplistic way. The focus of his research is not the actual cost functions but the policy decisions related to investments. He therefore recommends to investigate other cost structures, such as a return on investment based on utilisation of infrastructure and taking into account maintenance costs for decisions.

This research will provide an answer to the knowledge gap with respect to the growth/evolution of networks in a metropolitan area. This means that unlike previous research the network will need to evolve from a given origin/start state and then this evolution will be related to the demand and cost functions used. This research does not aim to come up with new demand-topology or cost-topology relationships but instead aims to provide insights in the underlying mechanisms that determine the shape and evolution of transport networks.

1.3.1. Contributions of this research

Previous Literature already shows that there has been extensive research into transport networks. From the previous research however some gaps in the existing literature and knowledge can be found. The very recent work of Saidi (2017) already mentions the severe lack of models for ring-radial structures. The previous research has not attempted to grow a ring-radial network, and relations between the network evolution and the influence of demand and cost functions on this evolution are not documented either. Earlier research did investigate the relations of demand functions as part of a sensitivity analysis. The focus of the research was not the relation between the network topology and the demand function, but the sensitivity of the performance of a topology based on a change in the demand function. The effects of transport modality and cost functions has only been researched in comparative scenarios or in research that assumes a fixed network and then focus on the modality effects. The comparison of this study with relevant existing works will be conducted in the literature study as part of the knowledge gap.

2

Literature Study

In this chapter, previous works on the relations between modal costs, demand and metropolitan network topology are discussed. These works can be divided into some categories. This section starts out with works on different transport modes and their competition in networks. Then, research in ring-radial networks (representing metropolitan areas) are discussed followed by discussing the main approaches in network theory and modelling network growth. Finally the knowledge gap is briefly mentioned at the end of the chapter.

2.1. Research on Modal competition within transport networks

One of the design decisions in urban transport policy is that of a transport technology for the system. Within the urban setting a choice between Light Rail Transit (LRT), Heavy Rail (HRT) and Bus Rapid Transit (BRT) Systems is commonly posed. All of these technologies differ from each other in terms of capital and operating costs, operational speed, optimal frequency, stop spacing and infrastructural requirements. As such various studies have been performed comparing all these modes. Research by Smith (1973); Dewees (1976) and Boyd et al. (1978) indicate that buses are more effective than trains in all situations, when comparing only the operator costs (Smith, 1973), only costs for users (Dewees, 1976) or when comparing for both user and operator costs (Boyd, Asher, and Wetzler, 1978). These results are generally explained by lower operator cost and/or access and waiting time costs provided by buses, which outweigh a potential advantage of speed from trains. Other research finds different results depending on the demand that has to be served.Allport (1981) found that based on only operating costs, bus is the mode with the lowest costs for low demand scenarios and that LRT is cheaper in the middle demand range, only shifting marginally in favour of bus when including user cost. More recently it lead to a parametric cost model developed by Bruun (2005) to compare operating costs for BRT and LRT finding conditions in which LRT dominates BRT. This usually comes from the difference in marginal costs for adding/removing capacity in peak hours. Tirachini et al. (2010) developed a framework to compare alternatives for a radio-centric urban network with radial lines from the borders to a central business district (CBD) attempting to minimise total costs, similar to Boyd et al. (1978) Their results provide insight to the characteristics of each of the modes competing in the radial network structure. Research by Badia, Estrada, and Robusté (2014) studies the formation of an ideal fully radial network based on the modal technology used and assuming a uniform demand. Those results provided insight into the mechanics and relationships between cost functions and physical characteristics of a mode (such as stop spacing) on the network topology.

2.2. Research on Ring Radial networks

With many real life city and metropolitan areas developing around a central core area or CBD it makes sense to have transport network models that follow this structure. Throughout literature there are several attempts to model transport networks and efficiency of urban transport networks (mostly metro lines or urban rail networks) using a radial network. Early research by Laporte et al. (1997) uses a radial urban network model with distinct core and an outer circular area to measure the effectiveness of various network topological shapes (Cartwheel, Start, U/Cross, Circumferential, Grid, Triangle, Half-Radial and Half wheel). Other research into various topological effects on urban mobility was conducted by Derrible and Kennedy (2009, 2010); Derrible (2012) who investigated 19 cities worldwide to demonstrate that network topologies play a key role increasing public ridership. By using cities from all over the world they eliminate cultural characteristics and city design influences. Their later research increases the amount of networks considered and indicators considered. Their study shows that a ring or circumferential transit line can greatly improve network connectivity, directness, and operational efficiency of the network. In a similar study Wang and Yang (2010) derived the benefits of a ring-radial structure from six Chinese cities showing that networks with "ring plus radials" can decrease the mean topological distance and better reflect operation efficiency of the whole network. Research by Vuchic (2005) classifies transit lines and discusses the advantages and disadvantages of radial and ring lines. Radial lines address the heaviest demands from suburban zones towards the core area or CBD where diametrical lines connect two (balanced) suburban zones passing this core/CBD zone. Ring lines are then connecting the radial lines creating opportunity for transfers and integrating the network by allowing shortcuts and robustness. In later research Vuchic mentions the important functions of ring lines: Improving connectivity among radial lines, making trips between radial lines shorter and serving the busy areas that usually develop in a ring around the core/CBD area. Early Research on radio-centric networks can be found in Wirasinghe, Hurdle, and Newell (1977); Wirasinghe and Ho (1982) who analysed radial bus systems for CBD commuters, Later within the research of Vaughan (1986) and Chen et al. (2015) who considered ring-radial transit systems by finding the optimal spacing of radial and ring lines. Similarly research by Tirachini, Hensher, and Jara-Díaz (2010) and Badia, Estrada, and Robusté (2014) use polar coordinate systems for optimising radial transit corridors. Latest research on radio-centric networks is conducted by Saidi et al. (2016b,a, 2017) who idealise the urban public transport networks of the Calgary LRT and Shanghai metro based on radio-centric networks with polar coordinates. The model by Saidi is one of the few rind-radial network models in existence that takes various cost effects into account as the model computes generalised transit passenger cost for a given network. However the model relies on static evaluation of this parameter and although Saidi claims this model can be used to evaluate network investment scenarios, in practise this only works with limited scenarios and detailed information. The model can thus only be used to compare a limited number of scenarios on the generalised passenger costs. It does not output the network topological indicator evolution or provide any insight into these characteristics.

2.3. Network Growth Models

The process in which networks evolve over time is an important aspect in the growing interest in network science. The network models that are probably most popular in the literature are random graphs based on the algorithms of Erdos and Rényi (1960) or Gilbert (1959), the small-world model from Watts and Strogatz (1998), and finally the scale-free networks based on preferential attachment by Barabási and Albert (1999). However,according to Dupuy (2013), graph theory-based studies often result in a static representation of the network, hindering the analysis of network evolution. Furthermore Ducruet and Beauguitte (2014) concluded that research concerning the evolution and dynamics of networks using network science concepts and methods has remained surprisingly unexplored as most studies adopt a static approach.

The modelling of transportation network growth and its history has been thoroughly reviewed by Xie and Levinson (2009) – Modelling the Growth of Transportation Networks: A Comprehensive Review. In their review they show progress within the field of modelling and analysing growth of transportation networks. According to Xie and Levinson (2009) the research studies into growing networks have followed five main streams: Transport geography, Optimisation and network design, Empirical models for network growth, Economics of network growth and Network science. Each of these streams covers a certain aspect of growing networks from a certain scientific background. For a better understanding of network growth models, the reader is referred to this extensive review.

In the years after this review by Xie and Levinson (2009) there have been new studies related to principles behind network growth.

Barthélemy and Flammini (2006) studied the effects of space on the structure of networks. In particular, the trade-off between node attractiveness (population size in a node) and costs (link lengths or distance) in a tree network were studied through the use of a growth model. In a further study Louf et al. (2013) studied the emergence of hierarchical structures in cost driven growth models for spatial models. Growing a tree shaped network based on costs within a random distributed node space resulted in specific cost ratio conditions for hierarchical (hub and spoke) structures to appear. These studies are only completed for tree shaped random node networks, and do not provide insight in networks where either hierarchy is not a factor or where the points are not randomly distributed over the space, like the metropolitan networks studied in this research.

Schultz et al. (2014) propose a growth model that describes how spatially embedded infrastructure networks might come to be. Their model consists of an initialisation phase, where the network grows in a tree shape based on cost minimisation, and then a growth phase where a trade-off exists between link redundancy and cost optimisation. Plietzsch et al. (2016) apply this specific model structure to analyse the trade-off between local and global redundancy in power grids. Although this application is very specific and does not show relevance to this study, the modelling method and processes are useful to model multiple decision criteria a decision maker might face.

Verma et al. (2016) take the network pruning model as an alternative approach to network growth. Starting from a fully connected network, the impact of a link utilisation threshold has been determined and found that some intermediate value for the threshold parameter results in core-periphery structures for networks. Although this research does not "grow" a network the approach is still relevant for evaluation of networks and performance.

Lems (2017) conducted a research where the focus is policy and the decisions made to grow a network. Within his research a model has been designed to evaluate the impacts of investment decisions on network topology. Being one of the few researches found to actually capture network evolution as function of inputs related to decisions faced by investors. The model describes a generic spatial network rather than a ring-radial structure. The focus of his research is not the actual cost functions but the policy related to it.

2.4. Knowledge Gap

As mentioned earlier the previous works still leave gaps with respect to certain areas of this study. Within the various fields and studies some of the knowledge gaps have been defined already, but a summary is provided below. The table on page 8 provides readers with a quick-scan overview of all relevant research.

The proposed research will provide an answer to the gap with respect to the growth/evolution of networks in a metropolitan area. This means that unlike previous research the network will need to evolve from a given origin/start state and then the evolution will be described as function of the demand and cost functions used. This research does not aim to come up with new demand-topology or cost-topology relationships but instead aims to provide insights in the underlying mechanisms that determine the evolution of transport networks.

Research	Type	Focus	Node Positions	Multiple Growth Principles	Growth Method	Objective Function	Constraints	Demand	Evaluation
Xie and Levinson (2009)		Network Comparison	Square Grid	No	None	N/A	N/A	N/A	N/A
Tirachini et al. (2010)	Economics of Network Growth	Modal Competition, Modal Comparison	Polar	No	None	Minimise Total Cost: User Costs and Operator Costs	Modal Restrictions	Cyclical, Fixed	Performance: (Cost/Demand)
Louf et al. (2013)	Network Science	Hierarchical structures in growing networks	Random, Uniform, Planar	No	Link Addition	Maximise profit	All Nodes Connected	Fixed	Threshold Values
Schultz et al. (2014)	Transport Geography Economics of Network Growth	Growth of spatially embedded networks, applied to powergrids	Random, Planar	No	Link Removal	Minimise Cost	All Nodes Connected	N/A	Threshold Values
Verma et al. (2016)	Network Science	Core-Periphery development conditions	Polar	Yes	Link Removal	Maximise profit	All Nodes Connected	Fixed	Threshold Values
Lems (2017)	Economics of Network Growth	Network Growth Investment Strategies	Random, Planar	Yes	Link Addition	Minimise Traveltime; Maximise robustness; Maximise Connected Population	Maximum Link Length Local Scope	Fixed	Topology Indicators
Saidi et al. (2017)	Economics of Network Growth	Network Comparison	Polar	No	None	Minimise Generalised Passenger Costs (GPC)	Topology Restrictions	Fixed	Comparison of GPC
This Research	**Network Science** **Economics of Network Growth**	**Metropolitan Transport Network Growth**	**Polar**	**yes**	**Link Addition**	**Cost Benefit Analyis** (CBA)	**Connected Sub-graph**	**Latent, Changing**	**Topology Indicators**

Table 2.1: Knowledge gap table

3

Model

This chapter describes the Model that has been created to relate demand and cost functions to the evolution of a metropolitan transport network via topological indicators. First a visual representation of all elements discussed is shown in section 3.1 after that the the scope of the modelling framework is defined in section 3.2 followed by a detailed description of all the elements in the model in section 3.3.

3.1. Conceptual Model

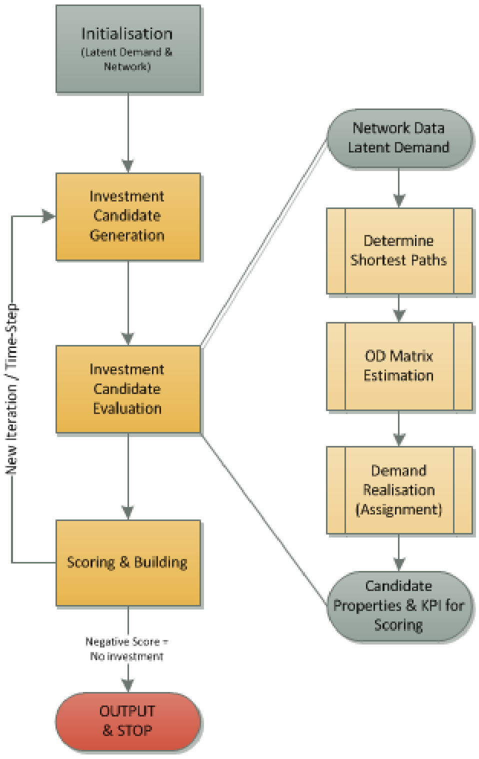


Figure 3.1: Conceptual Model

The major processes that can be distinguished in the model flowchart are the initialisation of the model in the input box, The iterative process of adding links to the network and evaluating the performance of the network (orange loop) until a stop criterion has been met (red stop). The evaluation process is detailed on the right showing the steps of the evaluation, from input to output. These outputs (properties & KPI) are stored for later evaluation.

3.2. Modelling Scope

The focus of this research,as mentioned in the Research Scope, will be the growth of monocentric urban public transport networks. More specifically idealised radio-centric networks, a simplification of Metropolitan Urban Public Transport Networks that exist in real life.

Modelling these idealised radio-centric networks will be done using a polar grid. A benefit of this is that the network shape of the model can be directly related to a radio-centric city and calculations can be simplified based on polar coordinates. Using a polar coordinate system and polar grid present a limitation for growth or connectivity within the network. Given the assumption that the graph is planar, links cannot cross each other. Furthermore the network can only consist of radials or ring(elements) limiting the solution space of the optimisation. This also means that unlike real world networks the computed network is limited in its allowed connections thus not all connections can and will be considered. This limitation can be linked to unimodality as non-planar links are commonly a higher or lower network level. Another limitation is the fact that the graph remains connected. This can be explained from theory, showing that the benefits of adding a node to an existing network given equal conditions is preferred over connecting any two other nodes that are disconnect from the original network. For users the benefits of being connected to an existing network mean they can travel to all destination nodes hence more latent demand is served and for operators the costs of construction are distributed over all origin-destination pairs served resulting in higher ridership and potential revenue.

Like in the research scope, the focus of the research is the emergence of public transport network infrastructure. In other words, the evolution of the infrastructural network is studied from any given initial network state until the network has matured and no improvements can be made given the predetermined criteria. This means that an initial link is specified and the network will evolve from that state. Any random link can be been selected, however due to the origin having many direct links and most of the shortest paths within the max planar graph going through the origin there is a central tendency towards the origin for at least the initial set-up phase given the limitation that the network is always a connected graph. These assumptions mean the network will always grow from either origin node or the end node of this initial link.

As mentioned in the introduction, investment strategy plays an important role in the evolution of public transport networks. Various investment strategies have been researched before and like some of these researches, the investments can be discretised in respect to time. Within this research a single investment can be considered every time-period. The choices for investments are either expansion of the network by means of new connections or improvements of the network by means of increasing capacity. This is a simplification of reality where investments are commonly a very complex decision where system benefits are compared for all kinds of alternatives. For each of these time-periods the most beneficial investment is selected based on a given objective function. This process is thus myopic, considering only the current state inputs and short term benefits. This explicitly rules out long term benefits a certain link might have. The objective function represents the decision-maker or the tool to evaluate alternatives for a decision-maker. The inputs and details of the objective function will be specified later but are related to real world decision-tools like cost benefit analysis.

Latent Demand is one of the input parameters for our model and will need to be given as a distribution for each node. This latent demand is assumed to be constant over time thus is not affected by network growth or any other factors. The latent demand can be defined in any way, but for this research the focus will be on distributions that only vary over the distance r to the origin. This is a simplification of often more complex demand calculations required to model a real world city, but this will still provide insight into the dynamics and relationship demand and evolution of metropolitan transport networks. The realised demand is dynamic and is dependant of the network realisation. This means accessibility plays an important role in the evolution of a network and that the growth of the network could be path dependant meaning that the investments done will influence future decisions. Although this can be explained from a real world perspective, the behaviour is a problem for comparing evolution and thus indicators might have to be adopted to exclude path-dependant effects.

3.2.1. Network Representation

The conceptual model uses a spatially embedded L-space representation of the network (Berche, von Ferber, Holovatch, and Holovatch, 2010). The network is represented by a simple, undirected graph using nodes as transfer stops / hubs and links to represent direct connections between transfer stops / hubs.

$$g \in N, E \tag{3.1}$$

Where:

$N_i = r \in [0, \mathbb{R}]$ and $\phi \in [0, 2\pi]$

$E = [N_i, N_j]$

A simple graph or strict graph Tutte (1998, p.2), is a graph that does not have any loops or multiple edges for the same node pair. An undirected graph is a graph in which edges have no orientation. The edge (N_i, N_j) is identical to the edge (N_j, N_i), i.e., they are not ordered pairs, but sets $\{N_i, N_j\}$ of nodes. This is a simplification and future modelling steps could add directional graphs if required. The graph is spatially embedded, meaning that all the node locations are fixed in space and thus link lengths matter. The graph is a planar graph meaning that links may not cross each other.

Networks will be built within a limited area using a finite predetermined number of nodes in order to limit the computational efforts required. The number of nodes can be increased at the cost of computation time. Due to node, link and network definitions it is not possible to build links outside of the pre-allocated geographical boundaries represented by the maximum radius R of the metropolitan area as this area determines the node locations and thus potential space links can span.Important parameters for the spatial component of the network are the parameters used to describe the spatial locations of the nodes and the underlying grid. The network is bounded by maximum radius: R, distance between radials: ϕ and the predetermined number of rings a network can contain: S Hence the underlying grid or full planar graph can be referred to as:

$$G \in R, \phi, S \tag{3.2}$$

Because of this limitation infrastructure can only be constructed between nodes. Given the network layout only radial lines (constant angle from origin) and ring lines (constant radius from origin) can be considered.

$$\forall E : r_i = r_j \parallel \phi_i = \phi_j \tag{3.3}$$

3.2.2. Modelling Assumptions

A number of assumptions is made on a more detailed level in order to simplify the model or to make the model applicable for our research goals.

- All Nodes are located on a polar grid with a maximum radius and a spacing between rings.
- Selected radius and rings are based on real world metropolitan areas and expert judgement.
- Population distributions have various functions but constant population
- Total population is chosen in the range of a larger European sized metropolitan area
- Average person makes about 3 trips per day (Ahern et al., 2013)
- Link lengths L are equal to the travelled distance between the nodes that a link connects.
- Only complete links are allowed.
- The network has to remain a connected network. Links can only connect new nodes to the existing tree/network structure, no disjoint networks are allowed.
- Link hierarchy is not considered.
- Investment,- maintenance,- and travel costs are assumed to be proportional to link length.
- The only investment decisions that are allowed are whether or not and where to build a link, or to add capacity to existing links.

- Disinvestment is not allowed. Thus, a network is only growing denser in terms of links.
- Access and egress times are not considered.
- Waiting time and transfers are not considered

3.2.3. Model Inputs

Two types of input can be distinguished. The first type are the intentional variables, the factors that are subject of this research, that change for every experiment. These are the independent variables of which the influence is studied and dependant variables of which the effects are measured.Examples of these variables are the Population Distribution function and the deterrence function for demand. Also factors for the operational costs are considered intentional variables here. Both these categories will be detailed more in the respective modules.

The second type are the extraneous variables, all other factors of influence that may influence the outcome but are not the subject of this study. These are kept constant during the experiments to prevent them of being a serious influence and the sensitivity of the final outcome is tested by a sensitivity analysis. Most of these variables apply to the initial network set-up. Some important secondary variables are the number of nodes (related to the number of rings (S), the distribution of nodes in space (related to the maximum radius (R) and the angle between radials(ϕ)) and factors like the Value of Time (VOT).

The main outputs of this model are key performance indicators These all depend on the order in which links are added to the network. So, as long as the growth model stores data on the added links and the order in which they are added, all output indicators can be calculated outside the growth model.

3.3. Model Building Blocks

3.3.1. Demand Module

Some of the blocks in the design are based on transport forecasting modelling. The steps of the classic four-stage model (Ortúzar and Willumsen, 2011) have been implemented in the design. The basic building block of the Four Step Travel Demand Module (FSTDM) contains the elements of Trip Generation, Trip Distribution, Modal Split and the Travel Assignment.

Trip Generation is applied in a simple form. The population of a node is assumed to be equal to the number of trips generated and for ease of calculations the number of trips attracted is equal to the number of trips generated: To generate trips a doubly constraint gravity model is used. This means that the number of trips between any two nodes is proportional to the product of the population in both nodes multiplied by some cost function. It is doubly constrained, in the sense that for any i the total number of trips from i predicted by the model always (mechanically, for any parameter values) equals the total number of trips from i (population P_i). Similarly, the total number of trips to j predicted by the model equals the total number of trips to j, for any j (P_j).

This results in a Trip Matrix with the following formula:

$$T_{ij} = K_i K_j P_i A_j F(C_{IJ}) \tag{3.4}$$

subject to

$$\sum_j T_{ij} = P_i, \sum_i T_{ij} = A_j \tag{3.5}$$

and using iterative balancing factors:

$$K_i = \frac{1}{\sum_j K_j T_j f(C_{ij})}, K_j = \frac{1}{\sum_i K_i T_i f(C_{ij})} \tag{3.6}$$

Where:
T_{ij} = Trips between origin i and destination j
P_i = Trips originating at i
A_j = Trips destined for j
C_{ij} = Travel Impedance between i and j
K_i, K_j = balancing factors solved iteratively.
F = Deterrence Function, function of the Travel cost (distance), describes the relative willingness to make a

trip as a function of the travel costs. From literature a vast number of functions have been suggested but the selected gravity model based on Ortúzar and Willumsen (2011):

$$F(C_{ij}) = C_{ij}^{0.5} e^{-0.25C_{ij}} \tag{3.7}$$

Because the gravity model (or its cost function) takes the travel resistance as function of distance the appropriate distance to use would be the experienced network travel distance experienced by the users. This in term would be the shortest path distance, however given a growing network the travel would only occur between already connected nodes, leaving no benefits for connecting new nodes, as travellers will then travel to those nodes that are further away in both polar distance as well as path distance. To avoid this pitfall, the polar distance is taken as simplification. The generated Origin Destination Matrix (OD-Matrix) must be interpreted as the trips generated in a fully connected graph. This effectively is the latent demand, the "potential **Trip Distribution**" that can be realised when connecting new nodes. Actual **Trip Generation and Distribution** will be performed based on relative accessibility, the actual demand serviced by the network.

Modal Split is implemented in the form of a Logit Model, widely used for in transportation forecasting in various forms, first theorised by McFadden (1973). Within the Logit Model the probability that a certain mode will be used is proportional to e raised to the utility over the sum of e raised to the sum of all utilities. see the equations below. Although widely used it is good to recall the assumptions of the logit choice model. First of all the Logit Choice Model has the property of having Independence of Irrelevant Alternatives. Simply put this means that if you add a mode, it will draw from existing modes in proportion to their existing shares. Furthermore the Logit Choice Model assumes that users and suppliers have perfect market information and thus can make an informed choice, however given the assumption of uncapacitated assignment this is not always a valid assumption. The effects however are limited as the model is not used for actual trip planning nor is congestion taken into account. The logit choice model is a deterministic choice model meaning that given the same conditions the choice will always be the same. This helps the model in some sense by allowing another research to obtain the same choice behaviour given the same parameters and allows for a fair comparison between the various inputs of our model, but might not account for random effects that occur in actual user choice behaviour. Last assumption is that switching between alternatives is cost-less. For our model this implies the user has access to both alternative transport and public transport, a fair assumption.

$$P_{PT} = \frac{e_{PT}^{U}}{(e_{PT}^{U} + e_{Alt}^{U})} \tag{3.8}$$

and

$$P_{Alt} = \frac{e_{Alt}^{U}}{(e_{PT}^{U} + e_{Alt}^{U})} \tag{3.9}$$

The alternative mode parameter can be specified in the model by setting its speed (V_{Alt}) Given the assumed operational speed of 25kph the assumed mode can either represent an existing lower level of PT (normal bus service) E-Bikes or car transit within the metropolitan area.

The sum of the probability of all modes will equal 1 effectively showing the Modal Split. The following steps are implemented for the Logit Model. First the exponential utilities for the alternative mode and the PT mode are computed. The utility is related to the (negative) travel time, meaning a lower travel time make a mode more likely to be selected. The actual modal split (likeliness in percentage) for both choices is then computed using the formulated mentioned above.

Assignment is also performed in a simple form. All-or-nothing (**AON**) assignment is applied, assuming that all travellers take the shortest path to their destination. This ignores travellers' preference heterogeneity in route choice, as well as capacity effects that may in practice cause people to choose different routes. Applied to the experiments in this study, the all-or-nothing assignment makes sense as long as the path lengths are very different. However, it assumes that operational parameters such as frequencies, presence and directness of lines and waiting times do not significantly impact travel times or route choice. The actual assignment effectively is a multiplication of the modal split with the latent demand matrix and then assigning those OD pairs to existing links irrelevant of the capacities.

3.3.2. Investment Module

The Investment Module like the demand module consists of a few steps. The first step is generating all options for investing in the network, so called candidates. After that the evaluation and scoring of these candidates is performed and finally the best scoring candidate (if exists) is implemented and the next period starts.

Candidate Generation

In order to generate investment candidates for expansion the network some definitions are required. First the current state of the network has to be defined. This is done using an adjacency matrix (A). In graph theory, an adjacency matrix is a square matrix used to represent a finite graph. The elements of the matrix indicate whether pairs of nodes are adjacent or not in the graph. For the simple graph with node set N, the adjacency matrix is a square |N| × |N| matrix A such that its element $A_{i,j}$ is one when there is an edge from vertex i to vertex j, and zero when there is no edge.

$$A_{i,j} = \begin{cases} = 1, & \text{if } i \text{ and } j \text{ are connected} \\ = 0, & \text{otherwise} \end{cases} \tag{3.10}$$

Within a Ring-Radial network not all connections are possible. Only edges that meet the $r_i = r_j \parallel \phi_i = \phi_j$ criterion are to be considered. The full planar graph G with this constraint will result in a fully connected graphs' adjacency matrix $M_{i,j}$ with a value of 1 if and only if an edge exists in this fully connected planar graph G

The Network Expansion Investment Candidate Matrix I can be constructed in the same way the adjacency matrix is defined. Its element $I_{i,j}$ is one if the edge has not been constructed yet and 0 if the edge has been constructed. Therefore elements that are non-zero indicate the links that can be considered for investment.

$$I_{i,j} = M_{i,j} - A_{i,j} \tag{3.11}$$

Given the unidirectional nature of links, only the upper triangular matrix has to be considered and duplicates can be removed in order to reduce the number of calculations required. (if $I_{i,j} == I_{j,i}$ then remove $I_{j.i}$)

Capacity Increase Candidates can be defined by adding additional capacity to an existing link. This means all constructed links are a candidate for expansion resulting in a candidate list of links denoted $H_{i,j}$

Candidate Properties

After generating the investment candidates the properties used for the scoring of these candidates are calculated. The relevant properties are:

- Length of the candidate edge (L), required to compute the investment costs for this candidate.
- Candidate Adjacency Matrix $N_{i,j}$, required to compute the impact on shortest paths
- Shortest Path Distance Matrix $D_{i,j}$, required to compute travel times
- Shortest Path matrix (SP), Resulting matrix of shortest paths with the implementation of this candidate link
- Resulting Trip Assignment in case the Candidate link has been constructed.

Scoring / Cost Benefit Analysis

Quah and Haldane (2007) defines a Cost–benefit analysis (CBA) as "*the systematic and analytical process of comparing benefits and costs in evaluating the desirability of a project or programme - often of a social nature.*" As such the CBA is fundamental to government decision making and is widely accepted as a formal technique for making informed decisions. It attempts to answer whether or not a proposed project is worthwhile. As stated earlier, the CBA can be applied to many fields of policy and is a common practise. Within this research the CBA is used to determine the best investment candidate. This is done by comparing benefits in terms of user-benefits (travel-time reduction) to the investment costs of operations. both are discounted for the investment time-horizon of 30 years.

3.4. Model Output: Topological and Performance Indicators

In this section, a choice is made on the output indicators that will be used to quantify and distinguish topological evolution of networks. In order to select topological indicators a short description is given of the topological indicators that are available. Then a selection will be made based on indicators that capture the evolution process of network growth as well as those that have some direct relation to the input parameters studied (demand and cost structures)

In order to measure performance the system can be evaluated for both the operators and the users perspective. For the operators the system operational costs are an indicator of the effectiveness of their operations. For the users the total travel time of all users (connected or not) is a measurement of the effectiveness of a network.

3.4.1. Network Topological indicators

Many indicators exist to describe and capture topological properties. These indicators range from very simple, requiring only a quick calculation with easily accessible network parameters, to very complex, requiring very complicated calculations. There are indicators can be calculated from the number of nodes and links, such as indices for connectivity, and simple network complexity (Haggett and Chorley, 1969). Other indicators quantify patterns like rings and webs (Xie and Levinson, 2007), or relate to shortest paths between nodes like the diameter and average shortest path lengths. The last group of indicators are used to describe nodal property distributions like the nodal degree, closeness centrality and betweenness centrality (Wasserman and Faust, 1994).

For this research the following Topological Indicators have been selected either based on ease of computations or on relevance for the studied input parameters:

The **beta** index (β) measures the connectivity relating the number of edges (E) to the number of nodes(N). The greater the value of β, the greater the connectivity. As transport networks develop and become more efficient, the value of β should rise as well. This indicator is both easy to calculate and gives some insight in the connectivity of the network, a user performance indicator:

$$\beta = \frac{E}{N}$$

The **gamma** index (γ) is another measure of the connectivity in a network. It is a measure of the ratio of the number of edges in a network (E) to the maximum number possible in the fully connected network(E_{max}) allowing to see how complete a evolved network is. This indicator increases linearly with every edge added until all edges are constructed, however capacity expansion does not increase this index.

$$\gamma = \frac{E}{E_{max}}$$

The **diameter** (δ) is defined as the length of the shortest path between the most distanced nodes of a graph. It measures the extent of a graph and the topological length between two nodes. In other words, a graph's diameter is the largest distance which must be traversed in order to travel from one vertex to another when paths which backtrack, detour, or loop are excluded from consideration. Note this takes the actual distance between i and j ($D_{i,j}$) into account, not just the number of edges that need to be traversed.

$$\delta = max_{i,j}(D(i,j))$$

The **Average Path Length** (APL) is defined as the average length of the shortest paths of all nodes in a graph. This is the sum of the shortest path between nodes i and j ($P_{i,j}$) and the total number of shortest paths in the graph (N_{paths}).

$$APL = \frac{\sum P_{i,j}}{N_{paths}}$$

The degree of a node in a network is the number of connections or edges the node has to other nodes. Given the graph is undirected then the Adjacency matrix A is symmetric and the degree of a node is the number of its connected neighbours given by: $k_i = \sum_j A_{ij}$. A simple indicator for a network topology can then be given by the **Average Degree** ($\langle k \rangle$) as function of edges E and number of nodes N:

$$\langle k \rangle = \frac{1}{N}\sum_i k_i = \frac{2E}{N}$$

The importance of a node is characterised by its centrality. There are many different centrality indicators such as the degree centrality, the closeness centrality and others, but we will focus here on the **betweenness centrality** $bc(i)$ which is defined as the fraction of shortest paths going through the node i ($\sigma_{st}(i)$) out of all shortest paths (σ_{st}). Like the degree its average is considered as indicator.

$$\langle bc(i) \rangle = \frac{1}{N}\sum_{s \neq t} \frac{\sigma_{st}(i)}{\sigma_{st}}$$

Ringness is an indicator of the fraction of rings that are constructed against the number of radials constructed.

$$\phi_{ring} = \frac{l_{ring}}{l_{tot}}$$

Where:l_{ring} is the total length of edges on rings and the denominator l_{tot} is the total length of all edges.

3.4.2. Performance Indicators

It makes sense to select indicators that provide insight into the performance of a network. Performance is measured along relationships that riders and operators might consider to evaluate the trips or service they respectively use or offer. Indicators that describe these relations often relate to experienced level of service as a ratio of costs. This can be either by monetising time or directly taking cost effects into account. Selected indicators that provide insight in the mechanics involved are detailed below.

A very basic network metrics that provide insight in the performance of the network is the **length** of the network. This can be directly related to all kinds of choices such as operational costs, travel impedance and all kinds of other metrics. The network Length (l_T) is defined as the sum of the length of all the constructed edges($d_E(e)$).

$$l_T = \sum_{e \in E} d_E(e)$$

The experienced **Total Travel Time** (TTT) is the sum of the Travel Time (TT_{ij}) between nodes i and j. This TT_{ij} is computed using a speed factor (β_{mode}) and the distance between nodes i and j (D_{ij}). In order to get the system weighted total travel time the TTT is multiplied by the latent demand (Q^*) to get the experienced total travel time. This also takes the modal split into account meaning that both the selected PT mode and the alternative mode are considered for the total travel time.

$$TTT = \sum_{ij} \beta_{mode} D_{i,j}$$

$$TTT_w = TTT * Q^*$$

Detour factor: Whenever a network is embedded in a two-dimensional space, at least two distances between pairs of nodes can be defined. The absolute shortest distance possible ($d_A(i,j)$) which would be realised in the fully connected planar graph and the 'route' distance ($d_R(i,j)$) from i to j by computing the distance along the shortest path between i and j. The **detour factor**(Q_{ij}) for this pair of nodes(i,j) is then given by:

$$Q_{ij} = \frac{d_R(i,j)}{d_A i,j}$$

Potential Ridership is a measurement to see how many travellers would like to use the Public Transport mode if unlimited capacity was available. The potential ridership thus indicates all travellers that based on the expected travel time provided by a mode (speed/distance) will travel. This is the sum of all travellers for PT:

$$PR = \sum_{ij} d_{mode}$$

Denied Ridership (DR) is a measurement to see how many travellers are not able to travel with PT even though they have chosen the PT transit mode. This is a measure to compare the Potential Ridership (PR) to the capacity a mode is able to provide (Cap). Again this is a summation over all OD pairs.

$$DR = \sum_{ij} PR - Cap$$

VolumeOverCapacity($\overline{VOC}$) is the ratio of passengers that wish to travel through a link and the actual capacity of this single link. It is therefore defined at link-level and the average value can give an indication of the network crowding.

$$\overline{VOC} = \frac{PR_{ij}}{Cap_{ij}}$$

4

Experiment Design

This chapter describes the set-up of the network experiments and the implementation of the model.

4.1. Implementation

The model from chapter 3 is implemented as a simulation model in MATLAB. The flowchart presented in Chapter 3.1 allows for a modular model that consists of a main driver file and multiple specific modules and functions that can be called whenever required allowing to add features to the model at any later stage.

4.1.1. Simulation Model Files

The Main File in the simulation model invokes all other functions and files. It consists of an initialisation phase where the spatial distribution of nodes is defined and the initial network (a single link) is created. In order to run the iterative loop of network growth some other parameters are determined like the max planar graph's adjacency matrix, distance matrix and shortest paths and a maximum time horizon is defined to ensure the model will stop at some point.

After this initialisation phase the model starts the main iterative loop process as outlined in the flowchart of chapter 3.1. Effectively the Main Driver File will start the iterative loop and in that process calls relevant functions from other files. This Main Driver File controls the flow of the model and thus is the most important in running. More details and code can be found in the Appendices.

The next step is the Candidate Generation, which takes place by calling a separate module. After that each of the investment candidates are evaluated and their properties are stored for later scoring. The score-function weights the various properties and ensures the most valuable investment is selected and then the builder function adds this investment to the current network and advances the time a step, essentially functioning as loop. The process of generating candidates is detailed in Section 3.3.2. The Current Adjacency Matrix is subtracted from the Max Planar Graph Adjacency Matrix resulting in all potential Candidates (CandidateMatrix). The actual candidate is a single link, hence we then convert the CandidateMatrix into a Candidate List by finding the non-zero elements and using the indices of these non-zero elements to determine the nodes between which the candidate link is added. Alternatively the Candidate for Investment is an increase in capacity. This means comparing the current link-load to some function of discomfort/dis-bonus for travellers leading to reduced benefits. The advantage of capacity related investment is that given the limitation of a single investment in a single time period. As with the main driver file, specific details on this function can be found in the Appendices

After the Candidates have been determined the next module can be called. the Candidate Evaluation Module calculates all relevant properties for the scoring and implementation of a candidate. This is handled by directly calling functions for separate processes. Each of this functions are split in separate files for ease of programming and later evaluation. Determining the Benefit Cost Ratio is the next step in the process. this actually performs all of the demand related loops and is the main relation between the implementation of the network and the potential addition of the links. Some other functions are used in the simulation model. These include the gravity model function, Utility Logit Function and the Shortest Path Function. The Gravity Model is used only in the initialisation, but the Shortest Path Function and Utility Logit function are called in every iteration.

4.2. Verification

In order to verify that the simulation model works as expected a number of tests can be conducted. These are divided into individual module tests and integration tests.

In the individual module test the separate modules and functions can are tested by defining specific inputs for a function, running the module and comparing the output of the function with the expected output based on selected inputs. This can be done because the model is modular and each section should run without problems before integrating it in a larger model. Integration testing is done whenever a module is verified and is done by combining modules or using the final model with parameters for which the behaviour can be easily predicted. The goal of integration testing is to ensure that all interactions between models are correct and no errors occur, as well as to ensure that the final product is in line with expectations. The actual checks for each module as well as integration tests can be found in Appendix B

4.3. Scenario design

4.3.1. Population Distribution

The generation of the latent demand is strongly related to the populations assigned to each of the nodes and the effects these population distributions have on the network evolution are subject of this research. In literature various demand functions on either a single dimension or a two dimensional plane have been utilised. This research will utilise similar simplified functions to represent the relationship between population and the distance to the central node. Following relevant literature (Berry et al., 1963; Casetti, 1967; Li et al., 2014; Badia et al., 2014), the following functions are considered: Uniform distribution, linear decay function (nodes further away from the core have less population) and exponential decay function (relatively steeper curve).

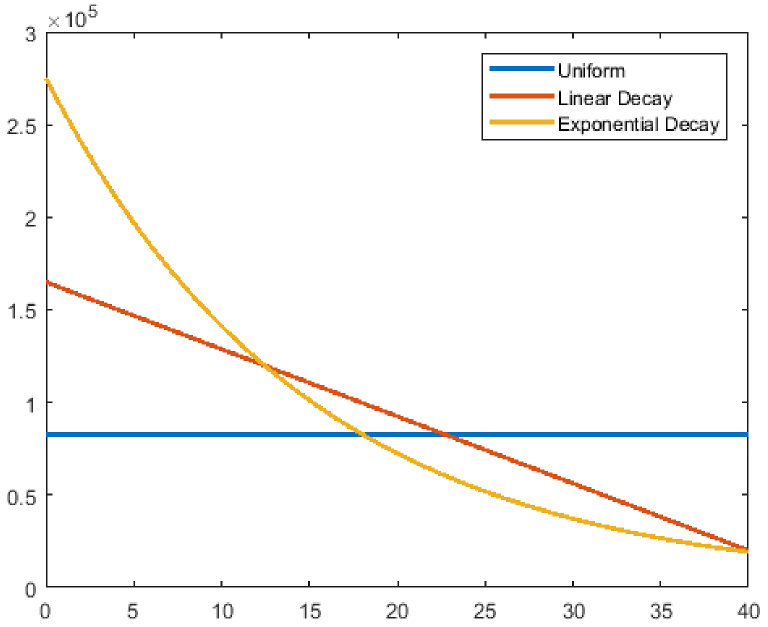


Figure 4.1: Various population distributions

To ensure the comparison between scenarios is fair and to ensure that population size does not impact the network evolution as such, the total world populations is assumed to be constant. This is achieved by ensuring that the integral under the selected population distributions is equal. Figure 4.1 shows distributions as function of r. The selected population size is roughly 8.000.000 (8 million) similar to London (Great Britain),Paris urban area (France), New York city proper (United States of America) or Moscow (Russia). (US Census Bureau, 2014; Office for National Statistics (ONS), 2017). Specific details for each of these population distribution functions are provided in their respective subsections. As is the determination of the population per node.

Uniform Distribution

As mentioned before the Uniform Distribution is the most basic distribution possible. Used by various authors (Mun et al. (2003); Chu and Tsai (2008); Badia et al. (2014)) this distribution excels in simplicity.In this scenario the population is distributed uniformly across the plane. This means that each node in our model will have the same population, and in order to keep population size constant for all scenarios this will be the Constant Population divided over the number of Nodes resulting in the Average Population (P_{avg})

$$N_{pop} = P_{avg}$$

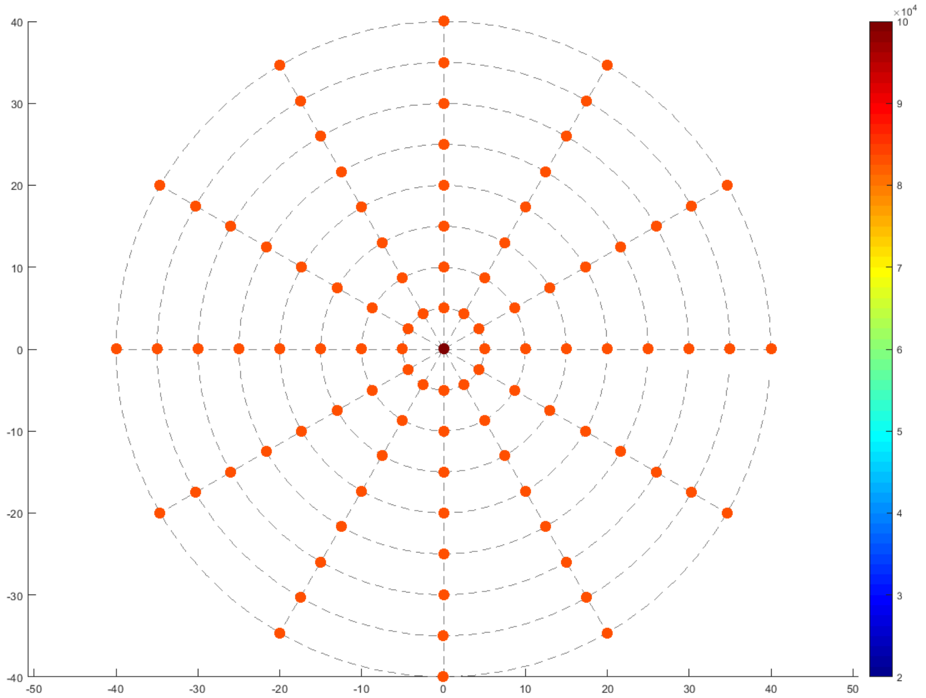


Figure 4.2: Uniform population distribution - Model

Expectation

The Uniform Distribution does not favour any specific area of the city and thus the only factor influencing the trips is the gravity model. The expected network is a network that has a load distribution related to the betweenness centrality of the Max Planar Graph Links. This relation obviously still depends on the combination of the costs for network investment (lower cost will mean more links will be constructed / upgraded) and the benefits a link will provide to users in reference to either the existing underlying network or the overload of a link.

Linear Relation to distance

The next distribution relates the density of populations in the nodes to the distance of the node to the central point of the grid. The closer to the city centre the higher the population or trip density most likely is (Berry et al., 1963; Casetti, 1967; Li et al., 2014; Badia et al., 2014). Boundary conditions set for this distribution where a minimum population on the edge of $P_{min} = 20000$ and given the mean population of $P_{avg} = 82500$ results in the maximum value at the centre of $P_{max} = 165000$ Function of the population thus is represented by:

$$N_{pop} = P_{max} - \frac{r}{r_{max}} * (P_{max} - P_{min})$$

This results in the following population distribution:

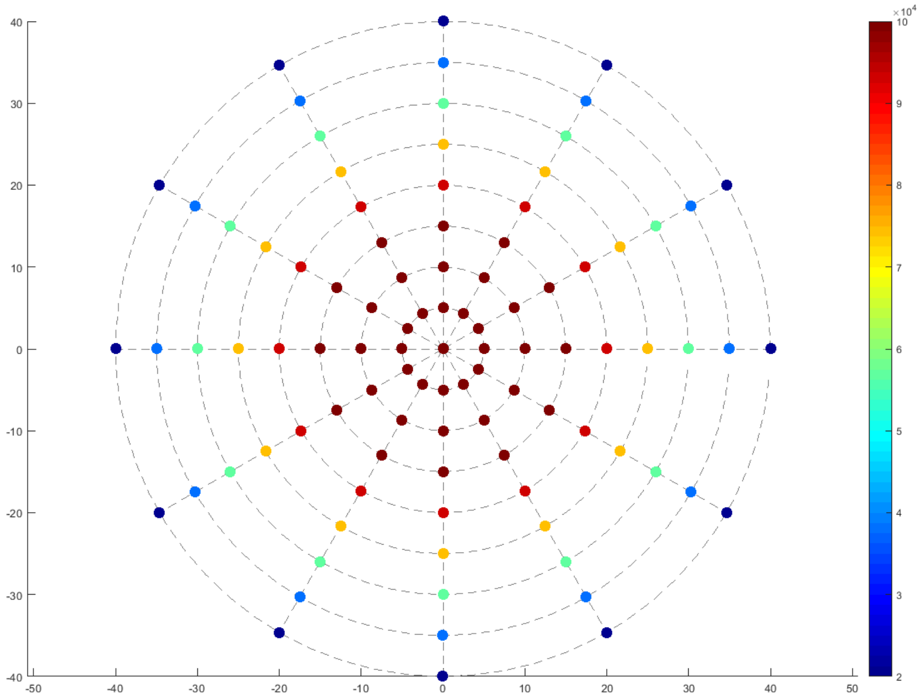


Figure 4.3: Linear decay population distribution - Model

In order to keep the total population equal given our non-linear node distribution (1 central node, 12 in all following rings) the linear relation is slightly different than might be expected and therefore the average value for this distribution is higher than for the uniform distribution.

Expectation

With higher populations assigned to the central zone the expected network development will favour connections closer to the origin of the network as the trips produced/attracted are directly related to the population density. The outer connections are expected to be omitted in the network building, depending on the parameters for operational costs, passenger value of time and the alternative mode characteristics.

Exponential Relation to distance

The relationship between distance and actual trips or population in a node does not have to be linear. Various studies using an exponential or power-law decay suggest that using an exponential function can mimic the behaviour of population or trip distribution over nodes really well.(Clark, 1951) Multiple cities with a strong urban core can be found in literature (Burdett et al., 2014) suggesting it is realistic to actually have a peaked population distribution.

$$N_{pop} = \alpha * e^{\frac{-r}{\beta}}$$

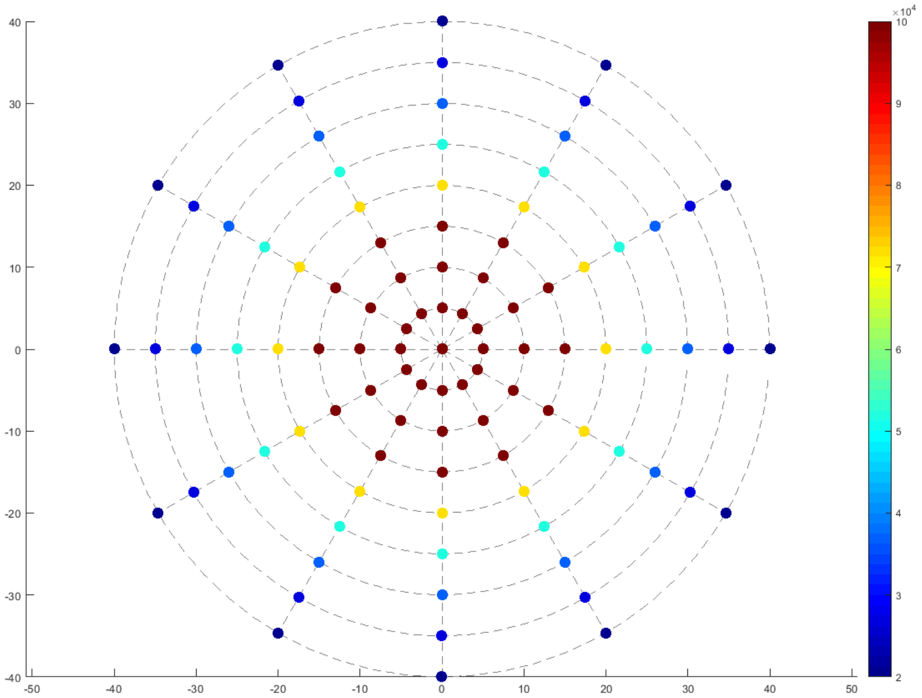


Figure 4.4: Exponential decay population distribution - Model

Expectation

With the very strong peak of population & Production/Attraction assigned to the central zone the expected network development will favour connections closer to the origin of the network as the trips produced/attracted are directly related to the population density. This also means the connections on the outer network rings might be completely neglected based on the choices for operational costs, passenger value of time and alternative mode characteristics.

4.3.2. Modal Cost Structures

Urban transportation can be classified by type. For this research only Public Transport Systems are considered and within these Public Transport systems a distinction is commonly made based on Right-of-Way, used technology and the operation type that is used. In literature (Vuchic, 2002) Three major categories are defined for the right of way / technology used.

- Rapid Transit (Metro) - Tracks/Lanes are physically separated. Its (mostly) electric rail vehicles are operated in trains and provide the highest performance mode of urban transportation.
- Light Rail Transit (LRT) - Operated on (partially) separated tracks/lanes, usually following Road Patterns. This Requires higher investment and has a higher performance than street transit. This can also include BRT systems if operated on separated tracks/lanes.
- Street Transit - Operated on urban streets with mixed traffic conditions, Street transit modes include mostly buses, trolleybuses and tramways/street-cars operated without separate infrastructure. This provides the lowest performance mode of urban transportation, at the lowest cost.

Transit system technology is one of the main transit system aspects to consider. Most people will know what a bus, trolleybus, tram, rapid transit or metro are. Less known is the fact that the Right of Way is the most important element in selecting a technology, because the right of way determines the performance and cost a given mode will have. The figure below shows the categories, specified by their "Right of Way(ROW)" category. Vertically the costs per unit length are shown, horizontally the achieved performance of the system in terms of capacity,speed and reliability

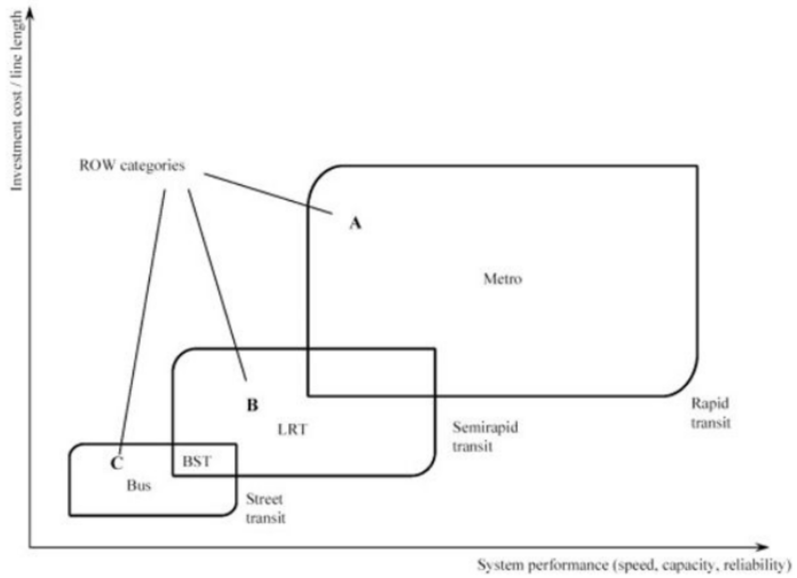


Figure 4.5: Transit modes compared by Vuchic

In this research these three generic types of transit will be considered and relevant parameters for each of these modes will be provided in the following sections as well as a general description of the mode in terms of technology used and parameter implications for the model.

Bus (Rapid) Transit System (BRT)

As mentioned by Vuchic (2002),Buses are one of the most widely used transit technologies in the world. Every city in the world that has any transit service operates some kind of buses. This is because a bus service is very easy to operate. All you need is a vehicle to carry (multiple) passengers, a garage/depot and some organisation of the service. Stops can be added anywhere and modified just as quickly. This means that buses are the most economical transit mode for lightly travelled lines. Bus transit is relatively labour intensive and lacks economy of scale effects: for every additional capacity unit, a new bus and driver have to be added. It is not possible to add cars like in rail systems.

On major urban corridors, which require faster, more reliable and higher capacity services than regular buses can offer, bus lines can be upgraded to offer higher level-of-service and higher capacity than regular bus lines. This type of service designated Bus Semi Rapid Transit or BST ("BRT"), represents a mode between regular bus and Light Rail Transit services. BRT investments are considerably higher than regular buses because they require construction of special lanes or roadways, stations and other equipment, however they are lower than Light Rail Transit because BRT Systems do not need electrification or tracks. Correspondingly, BRT performance and service, including speed, reliability and capacity, are higher than regular buses can offer, yet it does not match performance and level-of-service of Light Rail Transit because rail vehicle are more spacious, more comfortable and have better performance due to electric traction, permanent tracks and right of way give rail systems a stronger image for the public and investors.

BRT systems are created by provision of reserved lanes or roadways, preferential treatment at intersections, stops with multiple stopping locations which allow overtaking and simultaneous boarding of several buses, fare collection prior to boarding and other elements which increase speed and reliability of service. To increase line capacity, articulated and, in some cases with mostly straight corridors, double-articulated buses are used. An example of BRT can be seen in Figure 4.6, which shows the Bogotá BRT system with a station in place.

Figure 4.6: Bogotá BRT System (Photo by "Lorne Matalon")

Model Operational Cost Definitions

Capacity	[3.000 - 20.000 / Hour]
Maximum Capacity	[20.000 / Hour]
Infra Construction Costs	[3 - 20 mln/km / (**6 mln/km**)]
Operational Speed	[12-50 kph / (**35 kph**)]

Table 4.1: BRT Parameters

Light Rail Transit (LRT)

Light Rail Transit or LRT usually consists of spacious articulated cars operating in 1-4 car trains, mostly using reserved lanes or tracks, preferential treatment at intersections and usually some sort of overtaking opportunity at stops. Sometimes LRT has short tunnels in city centres or operates in pedestrian streets. The separation makes Light Rail Transit largely independent of traffic congestion, allowing them a higher operating speed and more reliable service compared to buses. Additionally Light Rail Transit benefits from a strong, attractive image and presence in many cities.

Light Rail Transit is a mode that is, by its performance and investment costs, lies somewhere in the range between bus and metro systems. Due to its diversity in physical and operational features, Light Rail Transit can be applied in many different locations and roles. The implementation of Light Rail Transit can be found on ground level in the form of trams, street cars, but similarly separate infrastructure can be found in many cases where the tracks are elevated or placed in a tunnel. Due to its diversity and ability to provide a much higher level of service than buses at a much lower investment cost than metro, Light Rail Transit is being built in more cities around the world than any other rail transit mode. Many cities in all areas of the world are presently building or planning Light Rail Transit systems. An example of a Light Rail Transit is shown in Figure 4.7 Showing the Light Rail in Jerusalem, Israel where a 2x5 unit vehicle is operated on separate infrastructure.

Figure 4.7: Jerusalem Lightrail Tram (nikischlicki)

Model Operational Cost Definitions

Capacity	[3.000 - 20.000 / Hour]
Maximum Capacity	[20.000 / Hour]
Infra Construction Costs	[10-30 mln/km / (**20 mln/km**)]
Operational Speed	[12-50 kph / (**45 kph**)]

Table 4.2: LRT Parameters

Rapid Transit or Metro

Rapid Transit (better known as metro) is a rail transit system with exclusive right of way (usually physically separated from other infrastructure). This exclusive right of way makes it possible to operate trains of up to 10 cars at maximum physically possible speeds between the stations. Metro systems usually have floor-level platforms and a fare collection away from the boarding area meaning metro trains can board and alight very large passenger volumes during a very short dwell time (as high as 40 passengers/second)contributing to metro's high throughput efficiency and travel speed.

Metro systems exist in about 100 cities throughout the world. In medium-size cities they commonly serve a few major corridors, in very large cities, metro is the backbone of transport providing services on an extensive network serving the entire city. Literature suggests that without metros, large cities could not exist in their present form, because no other mode could transport such large volumes of passengers as efficiently as metro. An example of the London Underground at Euston station can be seen in Figure 4.8

Figure 4.8: London Underground at Euston Station (Evening Standard 27 November 2017)

Model Operational Cost Definitions

Capacity	[20.000 - 80.000 / hour]
Maximum Capacity	[80.000 / hour]
Infra Construction Costs	[100-1000 mln / km / **(300 mln/km)**]
Operational Speed	[30-60 kph / **(60 kph)**]

Table 4.3: Metro Parameters

4.3.3. Scenario Overview Table

Below is a summary of all scenarios that will be investigated. The scenarios vary in either Demand Distribution or Transit Technology used. These combinations mean the effects of changing a single parameter can be investigated and the combinations of scenarios can lead to interesting results

Scenario Name	Demand Distribution Type	Transit Technology
BU - Uniform - BRT	Uniform	Bus Rapid Transit
BL - Linear Decay - BRT	Linear Decay	Bus Rapid Transit
BE - Exponential Decay - BRT	Exponential Decay	Bus Rapid Transit
LU - Uniform - LRT	Uniform	Light Rail Transit
LL - Linear Decay - LRT	Linear Decay	Light Rail Transit
LE - Exponential Decay - LRT	Exponential Decay	Light Rail Transit
MU - Uniform - Metro	Uniform	Rapid Transit
ML - Linear Decay - Metro	Linear Decay	Rapid Transit
ME - Exponential Decay - Metro	Exponential Decay	Rapid Transit

Table 4.4: Scenario List

5

Results & Analysis

5.1. Experiment Results

Each of the scenarios from the Experiment Section will produce results that can be split into two categories. Results that describe the indicators over the course of time, and results that describe the final state of the network where no investment is possible. After the results are presented, some analysis of the results will be performed in order to answer the research questions. These analysis will compare results of certain scenarios to find patterns in the data-set.

This section will start by presenting the final state Key Performance Indicator results and the graphs for the respective final states in section 5.1.1. After presenting these final state results, the respective scenarios will be discussed in section 5.1.2. This chapter then continues with a section that discusses the results in comparison to each-other in section: 5.2. After that a sensitivity analysis is performed in section 5.3 and the implications this research has for policy in section 5.5.

5.1.1. Final State Results

Key Performance Indicator Summary

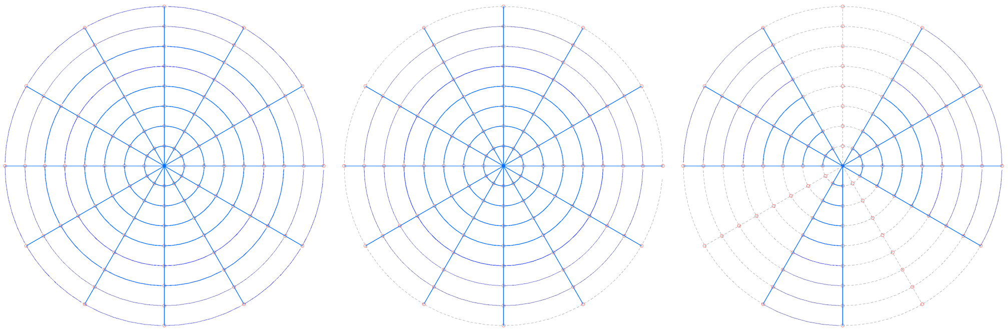
KPI	BU	BL	BE	LU	LL	LE	MU	ML	ME
Iterations	1689	1569	1010	1581	1485	1401	1208	1149	1041
β	1,9794	1,8557	1,2371	1,9794	1,8557	1,7320	1,4845	1,4845	1,3608
γ	1,0000	0,9375	0,6250	1,0000	0,9375	0,8750	0,7500	0,7500	0,6875
Diameter	80	80	80	80	80	80	80	80	80
Avg.SP Length	34,50	34,53	36,13	34,50	34,53	34,68	35,14	35,59	36,50
AvgDeg	3,96	3,71	2,47	3,96	3,71	3,46	2,97	2,97	2,72
AvgBC	0,0559	0,0560	0,0363	0,0559	0,0560	0,0567	0,0588	0,0608	0,0648
σDeg	0,8888	1,3067	1,6208	0,8888	1,3067	1,4147	1,4892	1,4892	1,4630
σBC	0,0465	0,0464	0,0447	0,0465	0,0464	0,0464	0,0481	0,0485	0,0518
Ringness	0,7020	0,6470	0,6110	0,7020	0,6470	0,5788	0,4597	0,3956	0,2820
System Length	1611	1360	925	1611	1360	1140	888	794	668
Total Travel Time	2699075	2786314	2928951	2396324	2475110	2405945	2170016	2222082	2181354
Detour	1,0000	1,0013	1,0635	1,0000	1,0013	1,0065	1,0255	1,0401	1,0756
Potential Ridership	4269771	4404054	2727212	4394750	4533197	4506656	4482076	4631270	4593854
Denied Ridership	4526939	5553432	3271504	2661376	3811578	4226265	-572934	565324	1805727
Avg.Link VoC	2,15	2,51	2,28	1,34	1,54	1,66	0,93	1,04	1,16
Accum.Score	113600	110376	67669	45125	43876	41607	3975	3922	3600

Table 5.1: Key Performance Indicator Summary

As can be seen in table 5.1 the final state indicators and metrics differ based on the input scenarios. Interesting results can be observed for various indicators:

In table 5.1 one can observe that the γ for Bu and LU scenarios is 1. This means the network is the fully connected MaxPlanarGraph, It can also be observed that more parameters for these scenarios are the same. Similarly the BL and LL scenarios have equal values for almost all the indicators. This would imply the resulting networks are the same. A closer inspection suggests there are some differences, mostly in available capacity. Inspection of the results clearly show that for a more peaked population the KPI clearly show that a less complete network is constructed. Similarly the effects of modes can be observed as a difference in mode commonly results in a different set of performance indicators. This can be explained from the underlying mechanisms driving the model: the BRT mode has a relatively low speed advantage to the alternative mode, meaning that in terms of generating the full network, benefits are just enough to warrant investment in the network, due to the low cost of the mode. For LRT the relatively higher speed mean the benefits are larger, partially because the speed difference between the mode and the alternative is bigger, combined with the relative distances in the network mean that the logit model flips so more of the latent demand will choose to use the PT mode. The still relatively low costs for the LRT mean that the resulting network is very close to the MaxPlanarGraph and positioned right between the BRT and Metro states. For Metro the relatively high costs mean that not all of the benefits can be obtained due to costs of connection being too high. This mechanism can be observed for the γ, Potential Ridership and System Length indicators. Another interesting observation is that the accumulated score is highest for BRT and lowest for Metro modes. This is mostly caused by the capacity expansion scoring relatively high in BRT, as a result of the potential ridership being far larger than the actual capacity of a link. This can also be observed in the resulting Average Link-Load that is highest for the BRT modes, and interestingly enough the ratio is higher for non-uniform modes than for Uniform distribution, an effect caused by the non-linear distribution of the population.

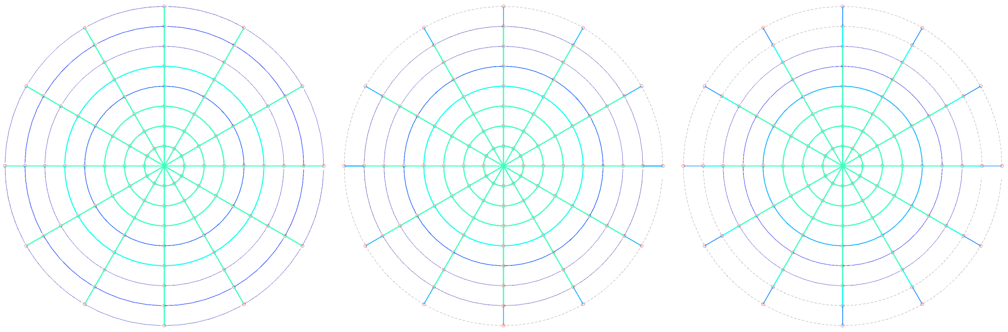
Final State Results - Final State Graphs - PT Passengers Realisation



(a) BU - BRT-Uniform

(b) BL - BRT-Linear

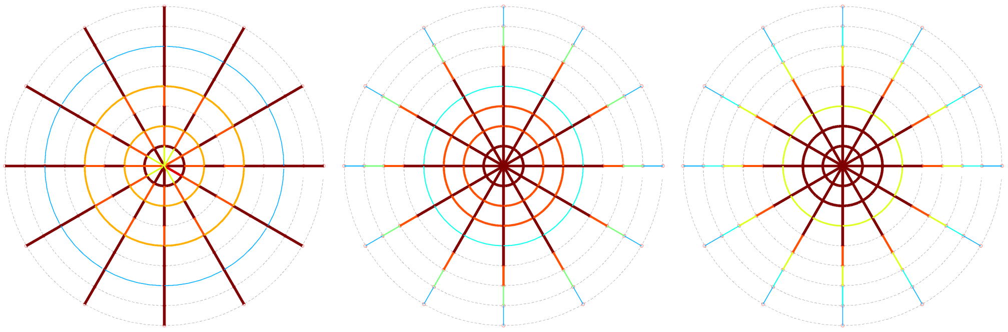
(c) BE - BRT-Exponential



(d) LU - LRT-Uniform

(e) LL - LRT-Linear

(f) LE - LRT-Exponential



(g) MU - Metro-Uniform

(h) ML - Metro-Linear

(i) ME - Metro-Exponential

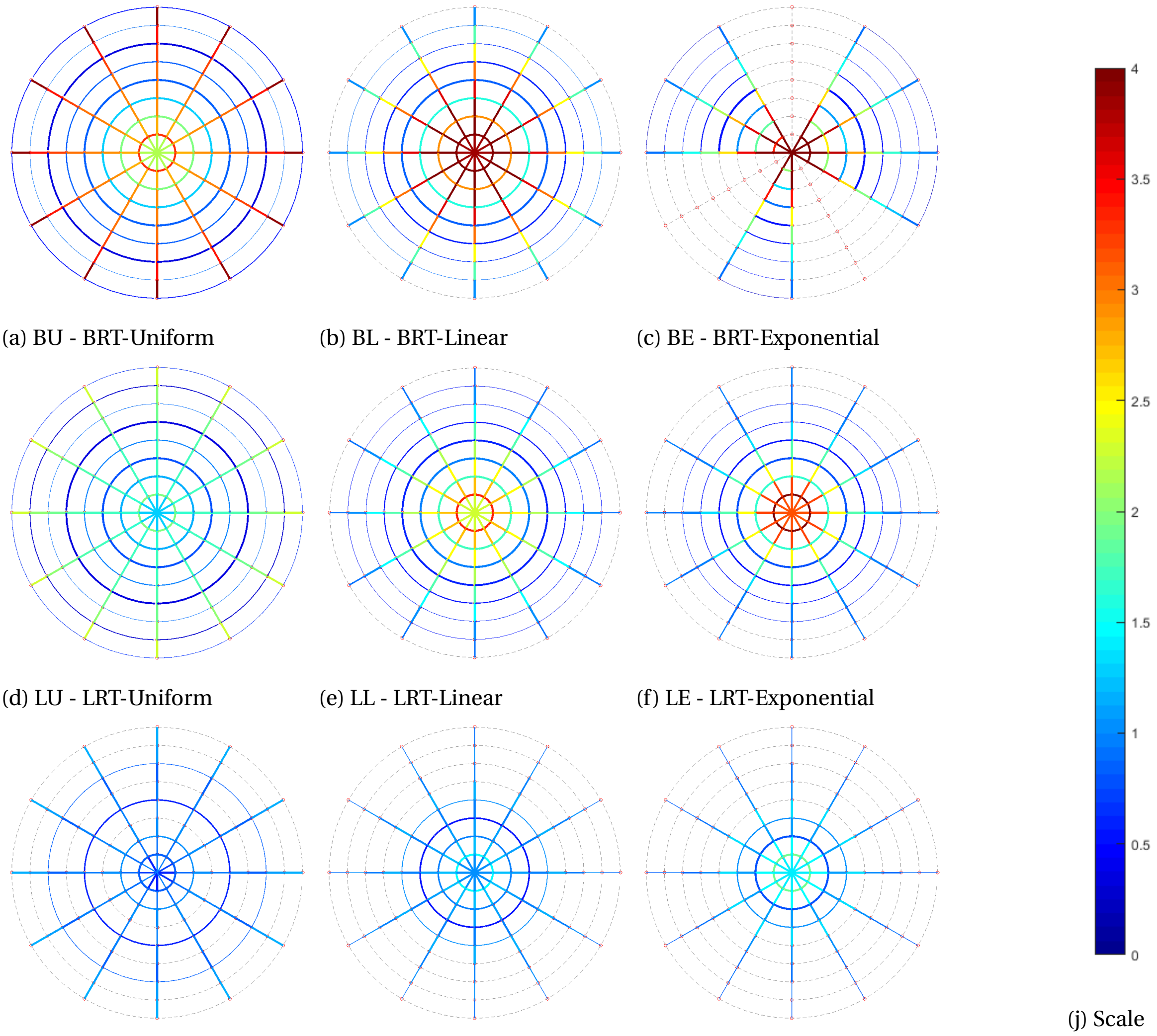
(j) Scale-bar

Figure 5.1: Finalised Network State for each of the scenarios. Colours of the graphs indicate total passengers transported along a link. This is the REALISATION of PT travel, not the demand

These graphs represent the final network states for the various scenarios. The scale-bar in 5.1j shows the ridership per link in peak hour. With higher maximum capacities for LRT and Metro modes it is clear that the BRT mode can only reach a limited capacity per link. LRT has a slightly higher maximum capacity and some differences in ridership can be observed between max capacity radials and slightly lower capacity rings (variations of blue). For Metro this is more clearly visible as the red/orange differences are more distinct and the final state graphs are a clearly affected by the various population distributions and modes. For population distributions one can observe that the more peaked the distribution (most clearly observable in ML/ME scenarios) the more the network limits itself to the inner core for higher capacities. This also holds for LL and LE cases and even BL and BE cases, but the scale prevents a clear distinction of the graphs. Similarly the Uniform Distribution shows that modes tend to connect outer edges/rings as well, due to higher demand making them viable investment options.

Final State Results - Final State Graphs - Volume over Capacity (VoC)

Graphs indicate the Usage(Passenger Volume) over Capacity for each final state.



(a) BU - BRT-Uniform (b) BL - BRT-Linear (c) BE - BRT-Exponential

(d) LU - LRT-Uniform (e) LL - LRT-Linear (f) LE - LRT-Exponential

(g) MU - Metro-Uniform (h) ML - Metro-Linear (i) ME - Metro-Exponential

(j) Scale

Figure 5.2: Finalised Network State for each of the scenarios. Colours of the graphs indicate VoC ratios in the respective final states

These graphs represent the final network state Volume over Capacity (VoC) ratio for the various scenarios. The scale in 5.2j shows the relative volume over capacity per link in peak hour (Potential Ridership / Capacity). The relative transported passengers are obviously different, but these graphs give an indication of the relative VoC for each of the links, and thus indicate links that are under stress and thus potentially could benefit from future investments. For most graphs the actual VoC is between 0 and 4. It can be observed that despite higher capacity for radials, the final VoC ratio is still higher on (inner)radial-lines. This is mostly due to more shortest paths being routed through these radials. Other high load links are commonly due to boundary effects: Outer edge radials sometimes have higher loads due to the boundary effect and gravity model. The VoC ratios are rarely larger than two as then a capacity expansion can be credited with the full benefits and usually the link is improved. All scenarios where the VoC is larger than this value seem to occur whenever the modal maximum capacity is reached. Additionally due to higher capacity per investment unit, the Metro VoC are relatively low and even below 1. Observable effects in the VoC ratios are thus mostly determined by the mode-parameters as the amount of people travelling is not affected by capacity (uncapacitated assignment) and thus the resulting capacity choices are directly related to the investment costs.

5.1.2. Individual Scenario Results

BU - Uniform Population & Bus Rapid Transit Service

For this scenario the population and thus trips are uniformly distributed for all nodes. This means all nodes have the same production/attraction for the Gravity model and the only limitation for trip generation is impedance of distance between nodes. The Bus Rapid Transit (BRT) is represented by defining mode-parameters: V_{mode}, Alpha (Cost), Capacity and MaxCapacity values based on the values found in literature, presented in 4.3.2. Resulting Key Performance Indicators and Network State can be seen in figure 5.3 and resulting final state network graphs in figure 5.4. A more detailed analysis of the results is performed in section 5.2.

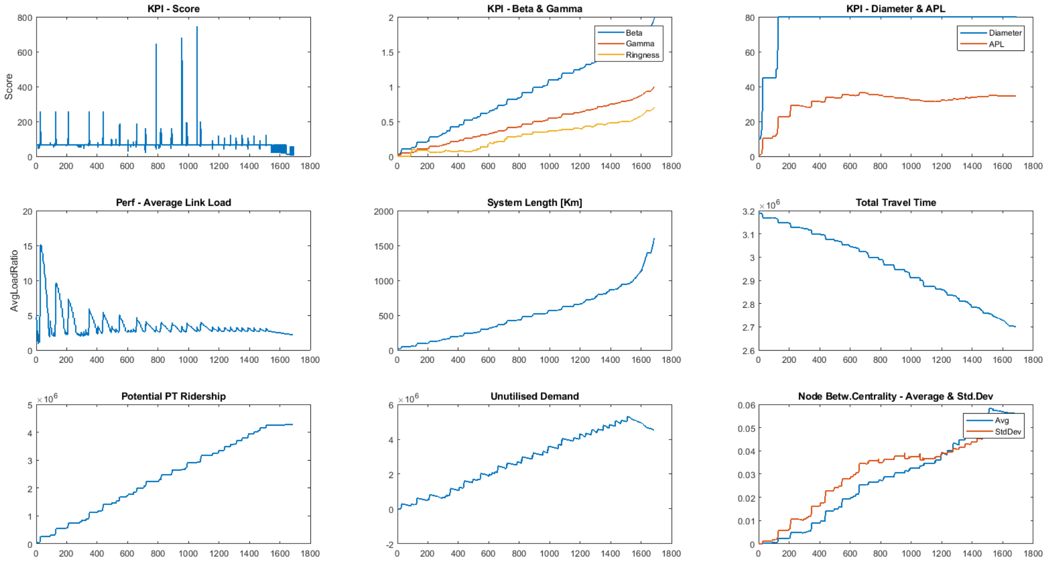
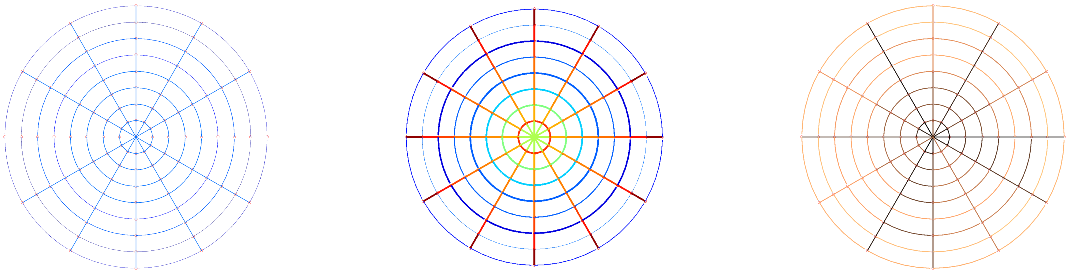


Figure 5.3: Key Performance Indicators for Uniform Demand and BRT Mode parameters



(a) Ridership for BU scenario (b) Link Loads for BU scenario (c) Link age for BU scenario

Figure 5.4: Final State Graphs for BU Scenario

BL - Linear Decaying Population & Bus Rapid Transit Service (BRT)

For this scenario the population are distributed with a decaying function in respect to the distance towards the central node as mentioned in 4.3.1. This means node have different production/attraction values for the Gravity model and the trip generation is thus limited by impedance of distance between nodes and the total production/attraction values of the nodes. Resulting Key Performance Indicators and Network State can be seen in figure 5.5 and 5.6. A more detailed analysis of the results in performed in section 5.2.

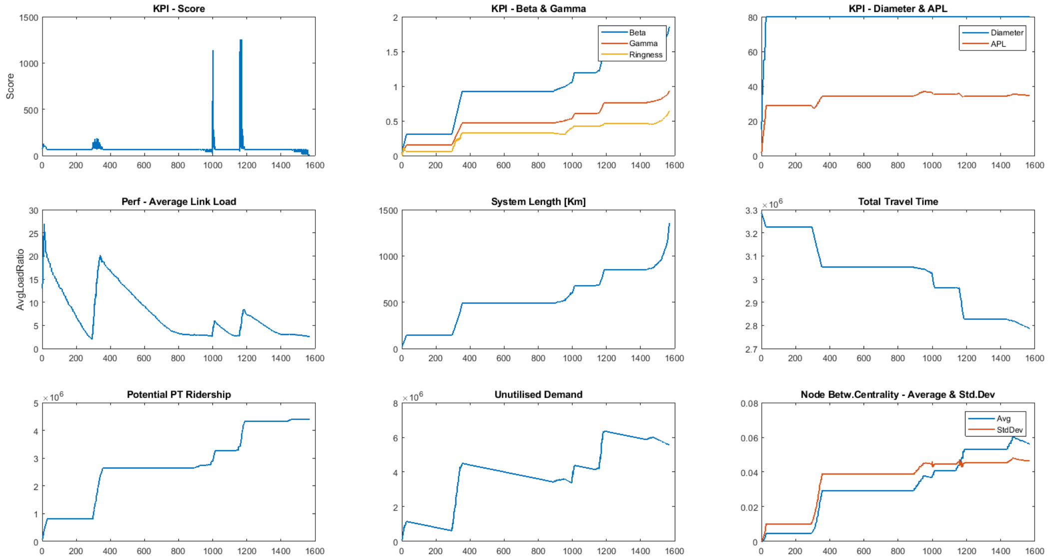
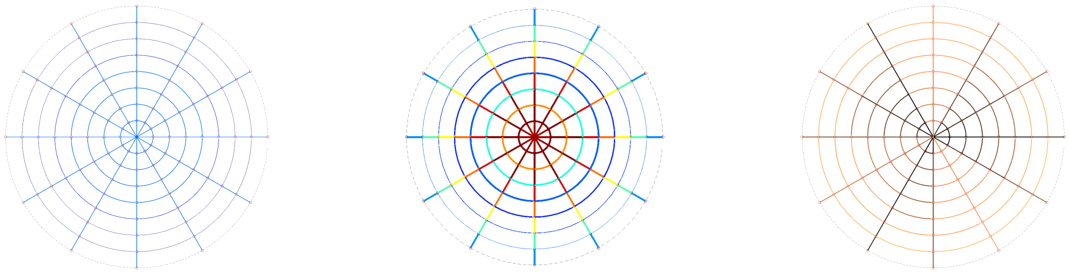


Figure 5.5: Key Performance Indicators for Uniform Demand and BRT Mode parameters



(a) Ridership for BL scenario (b) Link Loads for BL scenario (c) Link age for BL scenario

Figure 5.6: Final State Graphs for BL Scenario

BE - Exponential Decaying Population & Bus Rapid Transit Service (BRT)

For this scenario the population are distributed with a decaying function in respect to the distance towards the central node as mentioned in 4.3.1. This means node have different production/attraction values for the Gravity model and the trip generation is thus limited by impedance of distance between nodes and the total production/attraction values of the nodes. Resulting Key Performance Indicators and Network State can be seen in figure 5.7 and 5.8. A more detailed analysis of the results in performed in section 5.2.

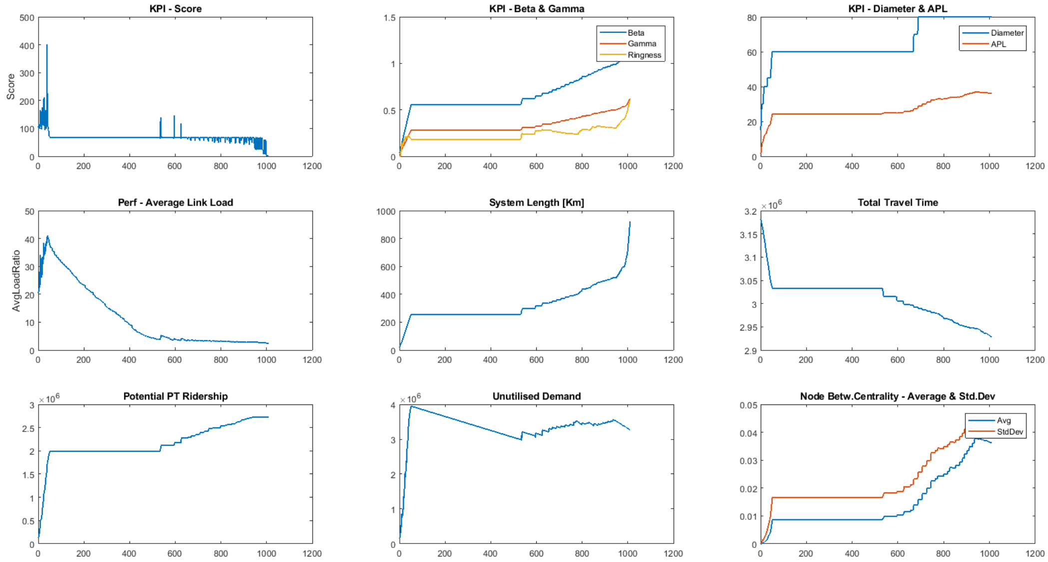
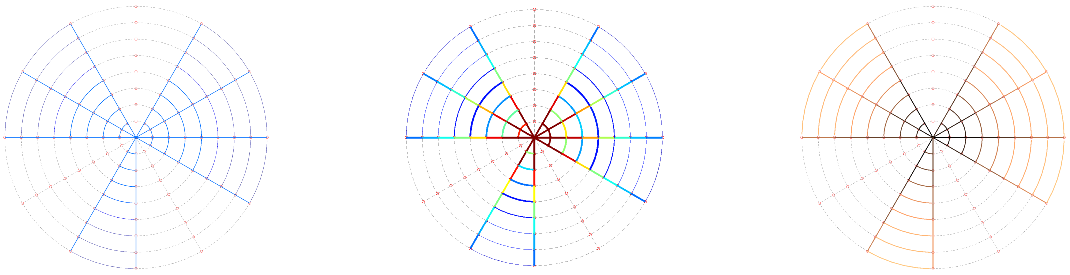


Figure 5.7: Key Performance Indicators for Uniform Demand and BRT Mode parameters



(a) Ridership for BE scenario (b) Link Loads for BE scenario (c) Link age for BE scenario

Figure 5.8: Final State Graphs for BE Scenario

LU - Uniform Population & Light Rail Transit Service

For this scenario the population and thus trips are uniformly distributed for all nodes. This means all nodes have the same production/attraction for the Gravity model and the only limitation for trip generation is impedance of distance between nodes. Resulting Key Performance Indicators and Network State can be seen in figure 5.9 and 5.10. A more detailed analysis of the results in performed in section 5.2.

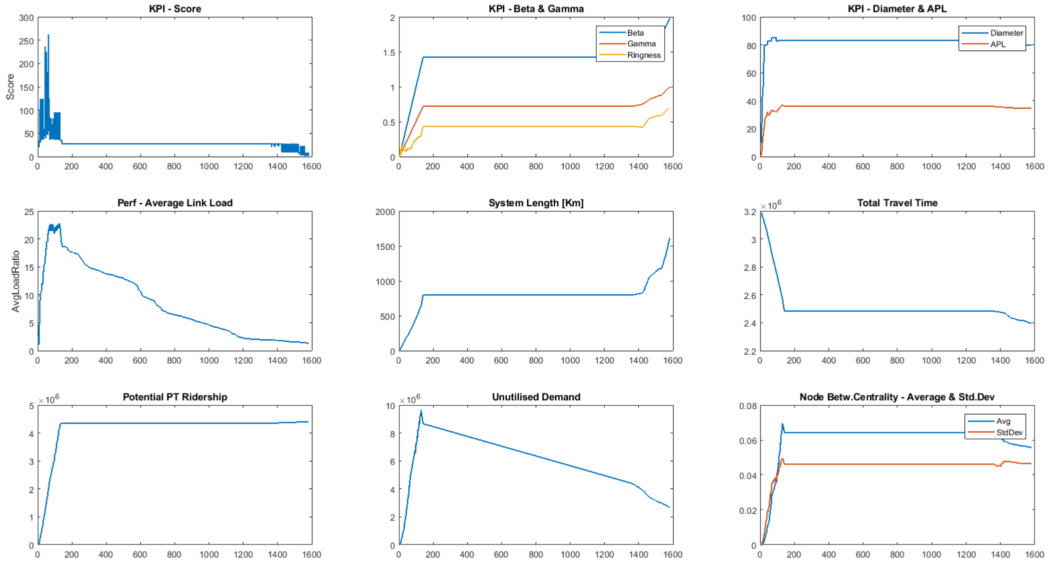
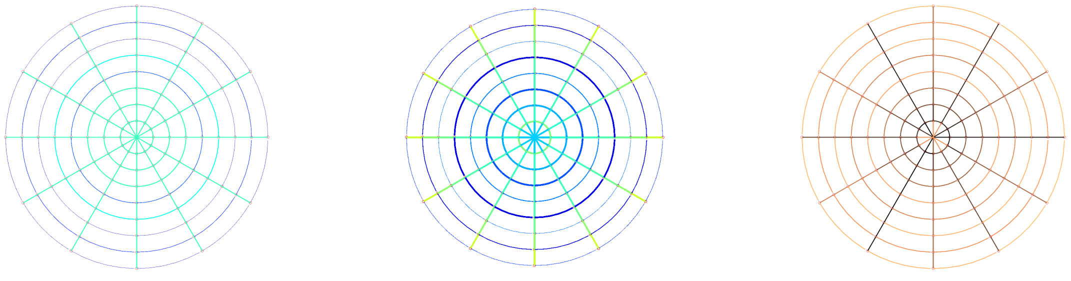


Figure 5.9: Key Performance Indicators for Uniform Demand and LRT Mode parameters



(a) Ridership for LU scenario (b) Link Loads for LU scenario (c) Link age for LU scenario

Figure 5.10: Final State Graphs for LU Scenario

LL - Linear Decaying Population & Light Rail Transit Service (LRT)

For this scenario the population are distributed with a decaying function in respect to the distance towards the central node as mentioned in 4.3.1. This means node have different production/attraction values for the Gravity model and the trip generation is thus limited by impedance of distance between nodes and the total production/attraction values of the nodes. Resulting Key Performance Indicators and Network State can be seen in figure 5.11 and 5.12. A more detailed analysis of the results in performed in section 5.2.

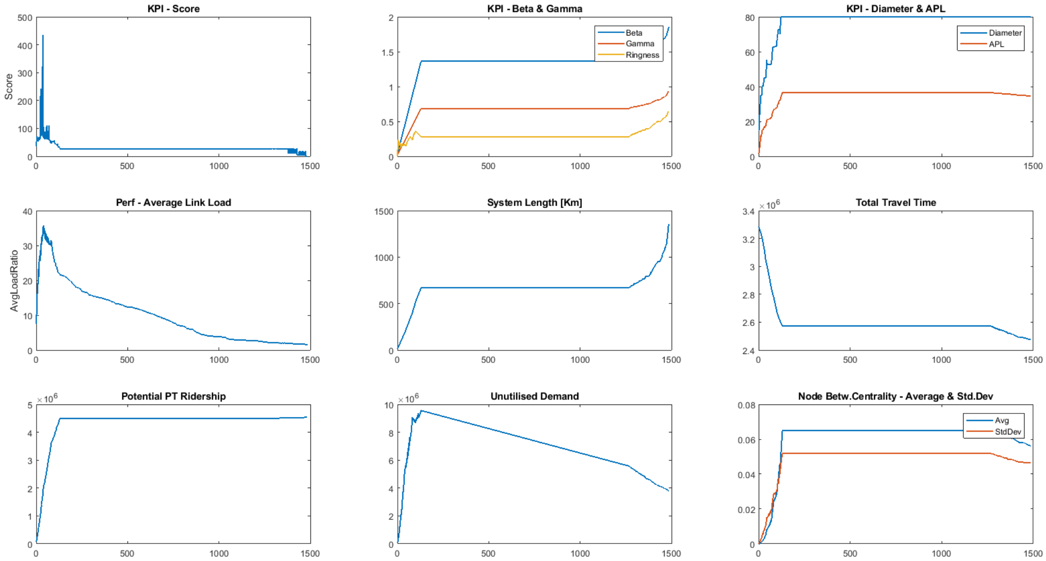
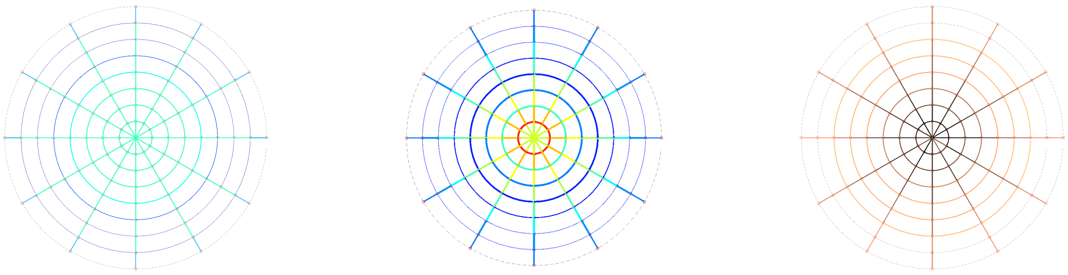


Figure 5.11: Key Performance Indicators for Uniform Demand and BRT Mode parameters



(a) Ridership for LL scenario (b) Link Loads for LL scenario (c) Link age for LL scenario

Figure 5.12: Final State Graphs for LU Scenario

LE - Exponential Decaying Population & Light Rail Transit Service (LRT)

For this scenario the population are distributed with a decaying function in respect to the distance towards the central node as mentioned in 4.3.1. This means node have different production/attraction values for the Gravity model and the trip generation is thus limited by impedance of distance between nodes and the total production/attraction values of the nodes. Resulting Key Performance Indicators and Network State can be seen in figure 5.13 and 5.14. A more detailed analysis of the results in performed in section 5.2.

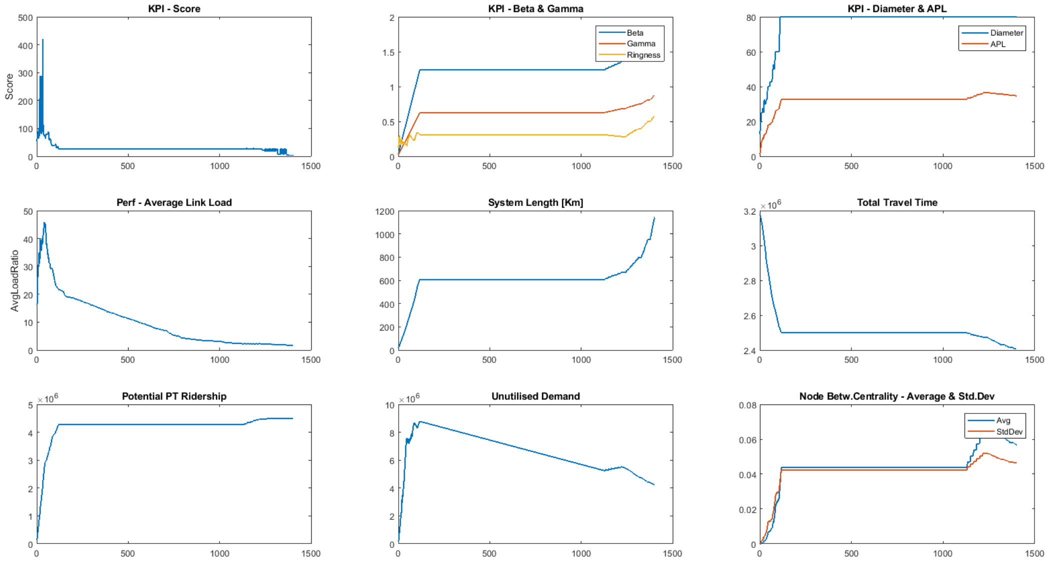
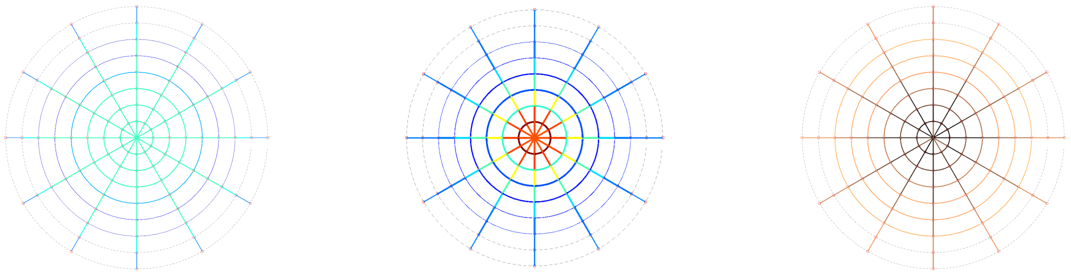


Figure 5.13: Key Performance Indicators for Uniform Demand and LRT Mode parameters



(a) Ridership for LE scenario

(b) Link Loads for LE scenario

(c) Link age for LE scenario

Figure 5.14: Final State Graphs for LU Scenario

MU - Uniform Population & Rapid Transit Service (Metro)

For this scenario the population and thus trips are uniformly distributed for all nodes. This means all nodes have the same production/attraction for the Gravity model and the only limitation for trip generation is impedance of distance between nodes. The Rapid Transit (Metro) is represented in the following parameters: V_{mode}, Alpha (Cost), Capacity and MaxCapacity values. The values for these parameters can be related to the values found in literature and presented in 4.3.2. Resulting Key Performance Indicators and Network State can be seen in figure 5.15 and 5.16. A more detailed analysis of the results in performed in section 5.2.

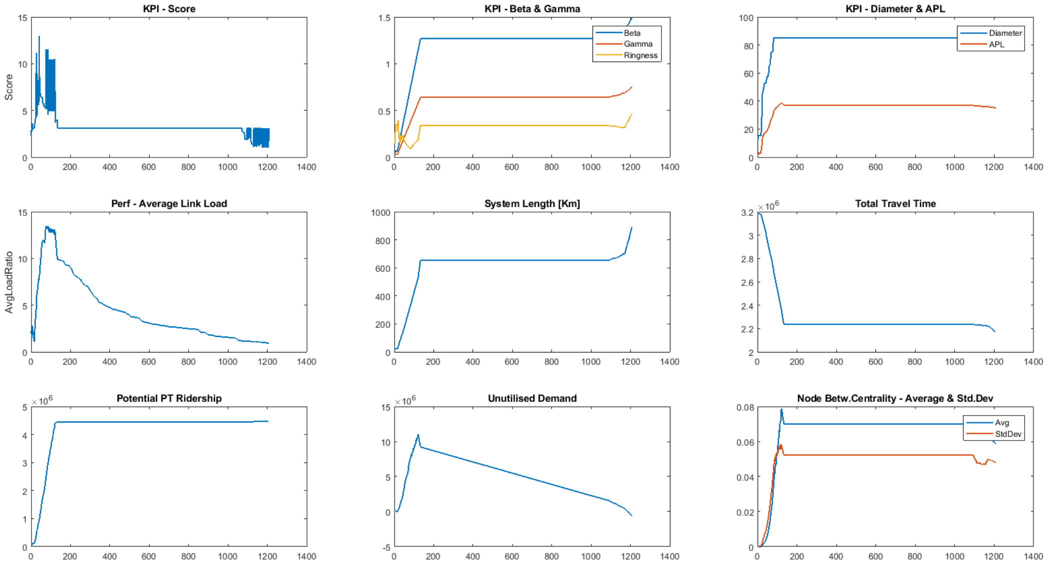
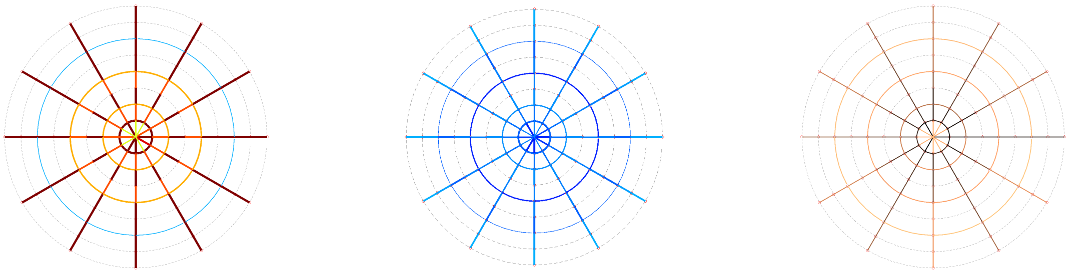


Figure 5.15: Key Performance Indicators for Uniform Demand and BRT Mode parameters



(a) Ridership for MU scenario

(b) Link Loads for MU scenario

(c) Link age for MU scenario

Figure 5.16: Final State Graphs for MU Scenario

ML - Linear Decaying Population & Rapid Transit Service (Metro)

For this scenario the population are distributed with a decaying function in respect to the distance towards the central node as mentioned in 4.3.1. This means node have different production/attraction values for the Gravity model and the trip generation is thus limited by impedance of distance between nodes and the total production/attraction values of the nodes. Resulting Key Performance Indicators and Network State can be seen in figure 5.17 and 5.18. A more detailed analysis of the results in performed in section 5.2.

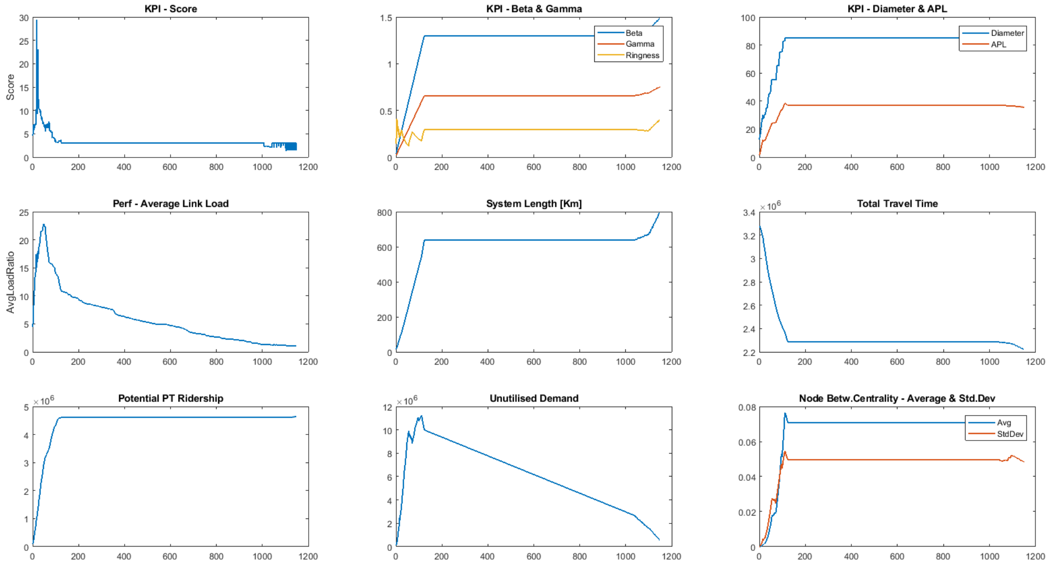
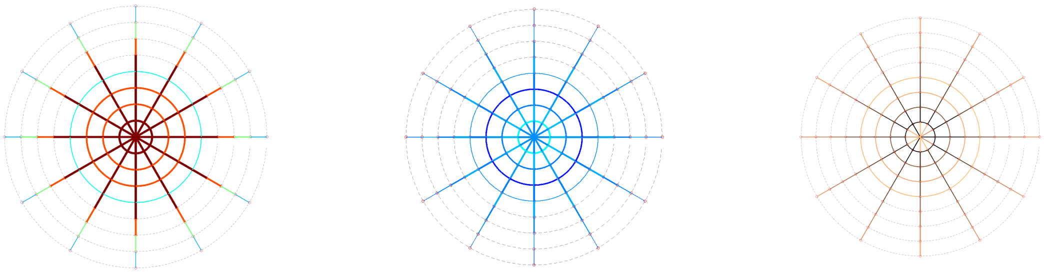


Figure 5.17: Key Performance Indicators for Uniform Demand and BRT Mode parameters



(a) Ridership for ML scenario

(b) Link Loads for ML scenario

(c) Link age for ML scenario

Figure 5.18: Final State Graphs for ML Scenario

ME - Exponential Decaying Population & Rapid Transit Service (Metro)

For this scenario the population are distributed with a decaying function in respect to the distance towards the central node as mentioned in 4.3.1. This means node have different production/attraction values for the Gravity model and the trip generation is thus limited by impedance of distance between nodes and the total production/attraction values of the nodes. Resulting Key Performance Indicators and Network State can be seen in figure 5.19 and 5.20. A more detailed analysis of the results in performed in section 5.2.

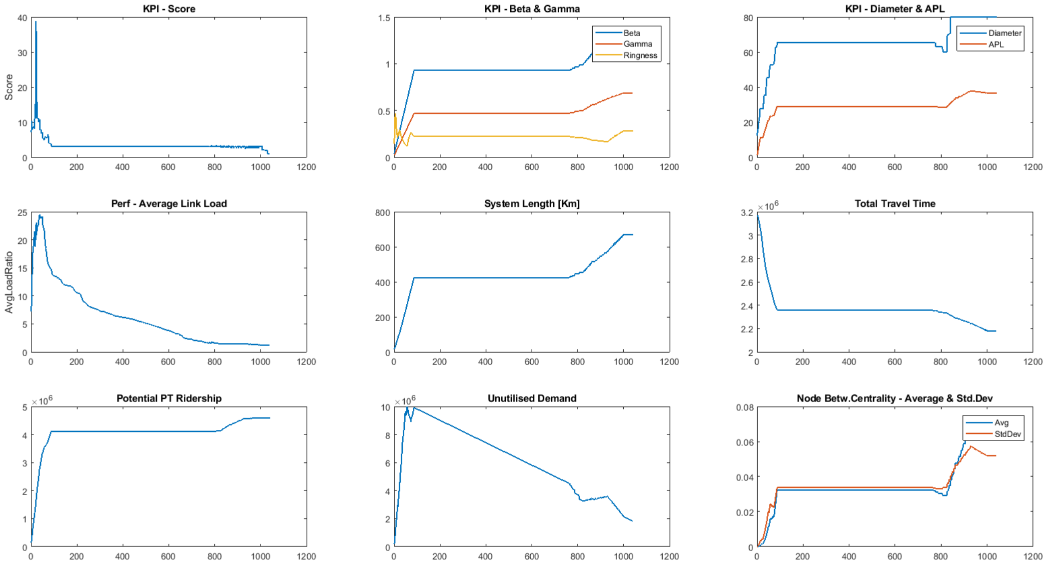
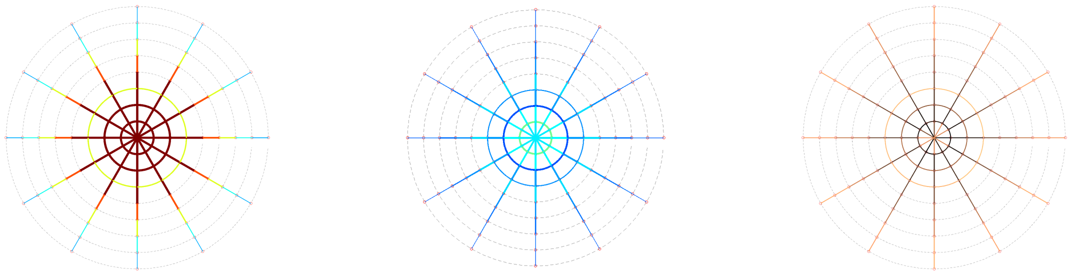


Figure 5.19: Key Performance Indicators for Uniform Demand and Metro Mode parameters



(a) Ridership for ME scenario (b) Link Loads for ME scenario (c) Link age for ME scenario

Figure 5.20: Final State Graphs for ME Scenario

5.2. Analysis of experiment results

In order to answer the research questions specific relationships between scenarios need to be analysed. This can be done for all the selected Key Performance indicators. For each of the indicators a separate analysis for the 9 scenarios will be performed in its respective subsection. For all these subsections the evolution of the KPI is shown for both Mode constant and varying population distributions and for a constant population distribution and having various modes plotted. This will help to provide insight in the behaviour of the indicator over time given the inputs of mode or population.

5.2.1. Scenario Discussion

As mentioned in each of the scenarios the individual results of the scenarios present some interesting results for discussion. In general we can see trends that will be discussed in a comparative manner by making graphs that plot the evolution of a given metric as function of either constant mode and varying distribution, or as constant distribution with varying modes. These relationships are more interesting and allow for a little more in-depth review of the metrics themselves. Individual remarks based on final states and evolution of metrics for each of the scenarios are mentioned below.

BU

As can be seen in 5.3 the development of a uniform BRT network is sort of a step-wise balance between expanding and adding new connections and increasing the capacity on existing links. One of the driving forces for this balance is the relatively high preferred usage / actual usage ratio. The final state graphs clearly show that the BRT network is able to span the full graph. This is likely due to the relatively low cost parameters and not due to the operational speeds, as those are relatively low. A sensitivity analysis for cost/speed effects can prove this theorem. The network can be classified as relatively high ringness indicating that due to lower investment costs the (minor)benefits of outer-rings are actually worth investing in.

BL

Similarly to the BU scenario in terms of advantages that costs and disadvantages of lower speeds. The resulting network has less connections on the outer edge due to the significantly lower demand at the edge resulting in a lower CBA ratio, thus not worth investing in anymore. otherwise the only large difference that can be observed is the different intensities of the various links, due to the higher population density in the centre, the link loads closer to the centre are higher then on the outskirts. The boundary edge effect cannot be observed as clearly due to the lower populations on the boundaries, but still exists

BE

Just like the BL scenario the population distribution's steeper drop in population means the network in the BE scenario is focused around the Central Business District even more, meaning the outer edges of the network are less developed. An interesting effect in the BE scenarios is the tri-axis network occurring. This is likely due to shortest paths and max detour someone is willing to take, with the addition of the relatively low operational speed not being able to bridge the gap effectively.

LU

The first observation is that the LU graph is actually the max planar graph. All potential links have been constructed and the resulting link-loads are relatively uniformly distributed. The most likely explanation for the full graph is that the higher speed of LRT mean that the difference in speed and thus travel time between the mode and the alternative is large enough to make all potential travellers choose for the PT mode (logit-model is effectively a comparison of relative travel time, once the difference is large enough, the value of logit flips) This increased benefit or potential travellers mean the full graph can and will be constructed by the algorithm. Another interesting observation is that initial expansion is followed by a phase of increasing capacity, finishing by connecting the missing links that have a relatively low CBA score due to the detour being relatively small and link loads under thresholds. The resulting shift in passengers can be observed in the denied passenger graph as the redistribution of the passengers causes a shift/dip in that sub-graph.

LL, LE

Similar to the LU scenario the parameters for LRT cause the graph to be the most largely connected graph resulting in a symmetric graph. Like the BL scenario, outer edges of the graph are no longer viable due to the lower demand between outer-edge nodes, no longer war-renting investments. Like the LL scenario the LE scenario limits the outer edge of investments to the inner core and density in the central core is higher than in the outer periphery. Due to capacity limitation the link loads in the inner core are obviously higher in the LE scenario, as can bee seen in the (b) sub-graphs.

MU, ML, ME

The graphs for Rapid Transport show a strong preference for Radials over Rings. This is most likely caused by the relatively high investment costs for Metro and the simple mathematical fact that rings are longer than radials. The resulting graphs for Metro scenarios are similar in metrics and only differ slightly in terms of capacity and radius of the outer ring. The resulting denied ridership is relatively low for this mode due to the high capacity it offers.

5.2.2. Network Metrics

In order to investigate the effects of population distribution or modal parameters have on the evolution of transport networks, various metrics and their values over the iterations are plotted against each-other. This helps to provide insight in effects that might be contributed to either of these inputs and quantify them.

Overall the trend in the observed metrics is an early phase of expansion of the network, followed by a period of capacity increments for all links and finally the remaining feasible network links are added. Not all indicators change with capacity hence for most indicators this pattern can be observed as a flat trajectory over time, indicating the network shape does not change, but the network capacity increases, this capacity increase can be observed in for example the denied passengers evolution. The following pages contain detailed graphs for the selected indicators and describe their paths.

Beta and Gamma

The β and γ in a ring-radial network are similar metrics. Similarly the (average) node degree follows the β index thus is removed in the comparison. Figure 5.21 shows the results for varying population distributions given a mode, figure 5.22 shows the results for varying modes given a population distribution.

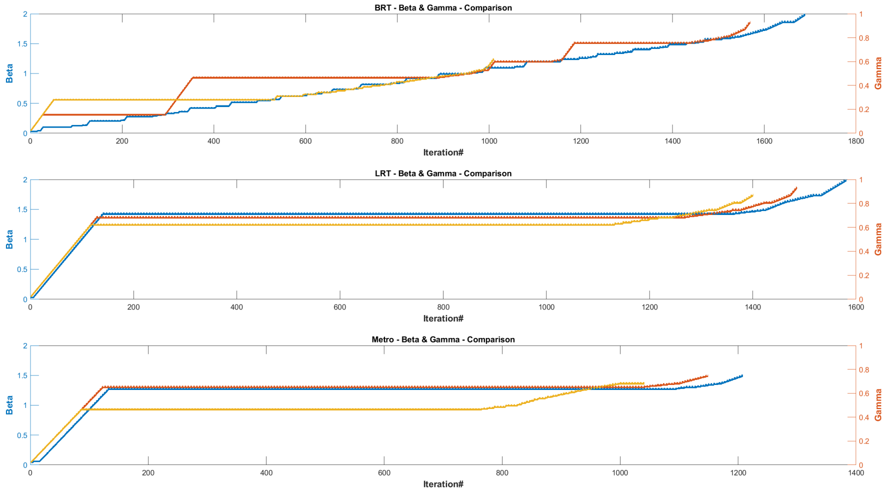


Figure 5.21: Comparison of β & γ for various population distributions given a certain mode, Blue for Uniform, Yellow for Linear Decay, Red for Exponential Decay

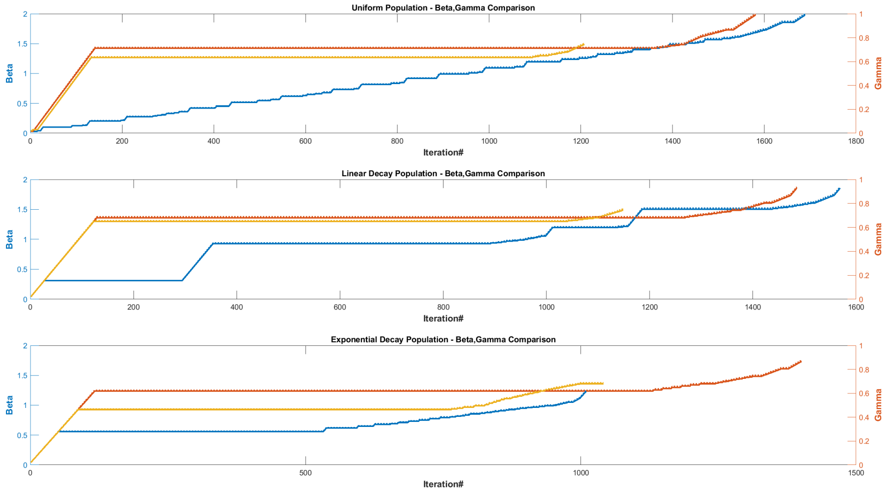


Figure 5.22: Comparison of β & γ for various modes given a population distribution, Blue for BRT, Yellow for LRT, Red for Metro

Looking at these graphs we can clearly see that the β and γ indicators are linked for these ring-radial networks as there is no situation where they significantly differ. This is most likely caused by the polar grid network and the definition of these metrics. In general a trend can be observed for a quick initial growth of the network in terms of new link connections, followed by a period of capacity increases until the final few links with relatively low demand are connected. This can be observed in almost all sub-figures. For Metro and LRT who have a relatively large speed over the alternative mode and sufficient initial capacity this behaviour is most clearly observable (figure 5.21). Similarly this holds for any population distribution although the magnitude of the indicators vary, the β and γ are relatively constant for the middle phase of the network growth in the model, indicating that increasing capacity is valued less than expansion for those scenarios. The BRT mode has a specific path for all population distributions. Its uniform distribution is a exact balance between the capacity increments and the expansions. The linear decay is characterised by an earlier capacity increase (inner core) followed by a secondary expansion and the exponential decay population clearly shows that for BRT the second expansion is not realised.

Diameter

The Diameter δ is a network indicator that shows the longest shortest path. For the ring-radial network the expected longest shortest path, as the name suggests,the actual diameter of the network grid. Figure 5.23 shows the results for varying population distributions given a mode, figure 5.24 shows the results for varying modes given a population distribution.

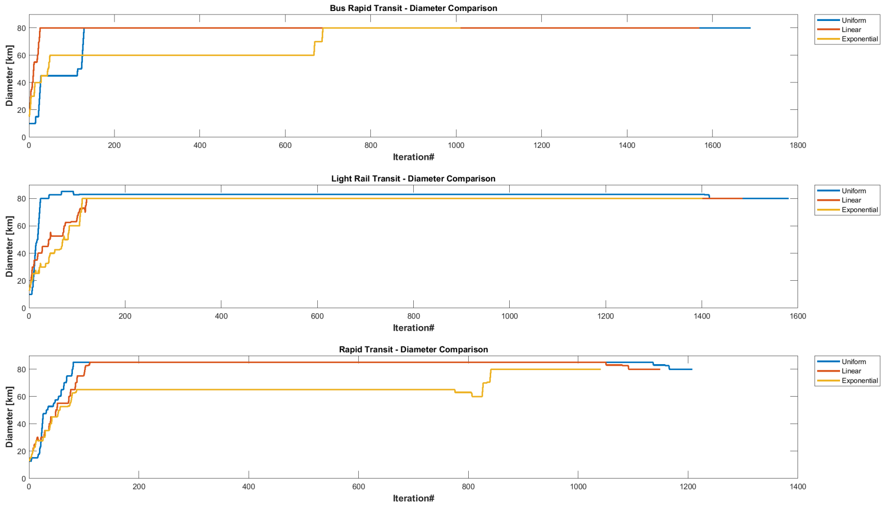


Figure 5.23: Comparison of Diameter for various population distributions given a certain mode

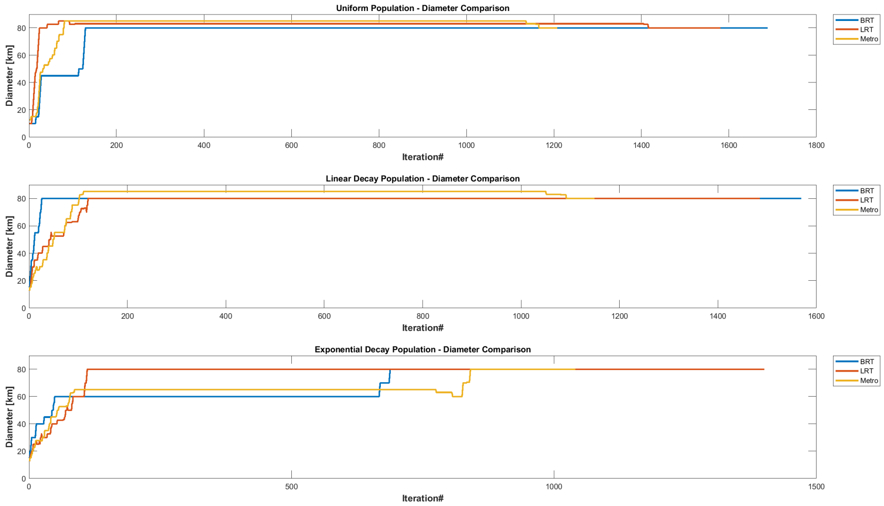


Figure 5.24: Comparison of Diameter for various transit-modes given a population distribution

For the diameter the final state for all modes and scenarios is the actual diameter of the network grid. For the Rapid Transit case one can observe the diameter to be larger than the network grid diameter for a long period of time (see lowest sub-figure of figure 5.23) this is because a closing link for the longest shortest path is added only in the final state. It can clearly be observed that for all modes and all population distributions the behaviour of this indicator is relatively similar for the all sub-graphs. At the same time it can be observed that the diameter for BRT moves to the steady state at the slowest rate due to the need of capacity increments between expansions. The initial phase (first 100-200 iterations) the indicators do vary showing that for the expansion phase the paths and choices are different for each scenario. For LRT and Metro the Uniform scenario logically leads to a larger initial diameter followed by the linear decay and the exponential decaying populations. When looking at population distributions we initially find that in decaying populations BRT initially has the highest diameter ad that for the uniform case this in fact reversed. This might be due to the population distribution peak value and average value and the location of nodes in the network.

Average Shortest Path Length (APL)

Just like Diameter this network indicator is related to the shortest paths. Instead of taking a single extreme value, this indicator averages all the existing shortest paths. Figure 5.25 shows the results for varying population distributions given a mode, figure 5.26 shows the results for varying modes given a population distribution.

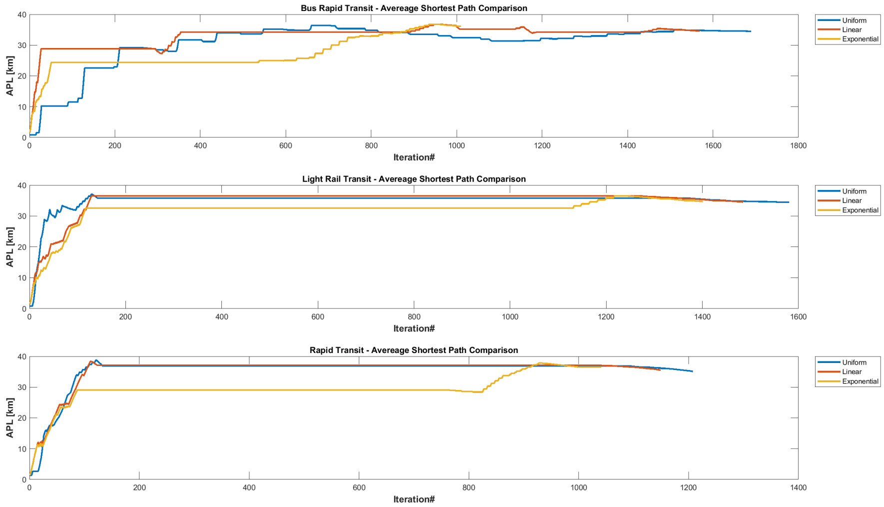


Figure 5.25: Comparison of Average Shortest Path Length for various population distributions given a certain mode

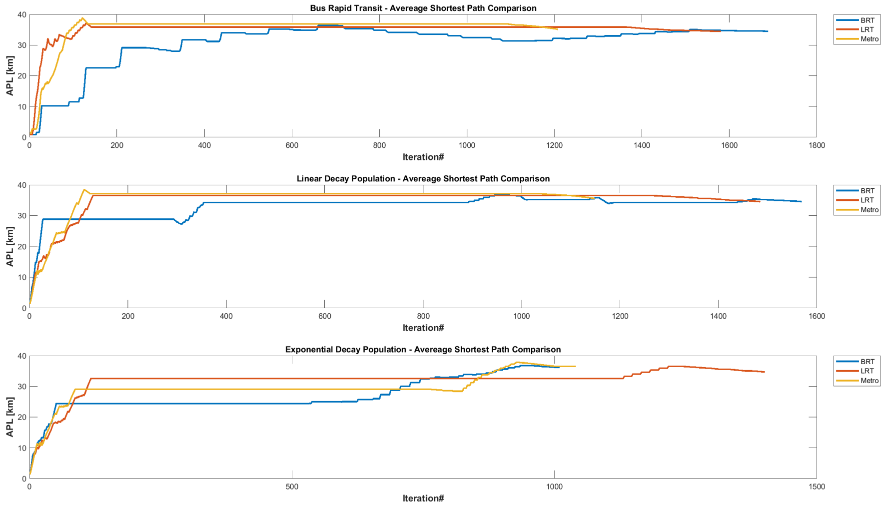


Figure 5.26: Comparison of Average Shortest Path Length for various transit-modes given a population distribution

From these graphs one can again observe that the increase of capacity does not affect the shortest path average as the path itself remains constant, only its capacity increases. Initially a violent phase of expansion results in strongly fluctuating shortest paths as new nodes(expansion) increase the shortest path for a while until the shortest path is either improved by a ring or missing radial is added. The average shortest path between nodes eventually approaches a stable point and for most graphs this is the average shortest path value for the Max Planar Graph of 34.4974. A larger value would indicate for Ring-Radial grid that some important links are missing for shortest paths resulting in sub-ideal routes. It can be observed that the population distribution affects this as for example the exponential distribution clearly has a lower APL indicating that the outer edges of the network are not connected and thus paths are relatively shorter. When comparing the modes it can be observed that BRT as always has a distinct different path and LRT and Metro follow similar patterns meaning that a significant increase in operational costs does not seem to impact behaviour for most scenarios.

Average Node Betweenness Centrality

This Indicator is more interesting for the node level. Instead of looking at the whole network the betweenness centrality. The indicator relates the number of shortest paths going through a node with the amount of shortest paths in total, giving an indication of node importance. The average value of this indicator over time gives an indication of the complexity of the network.

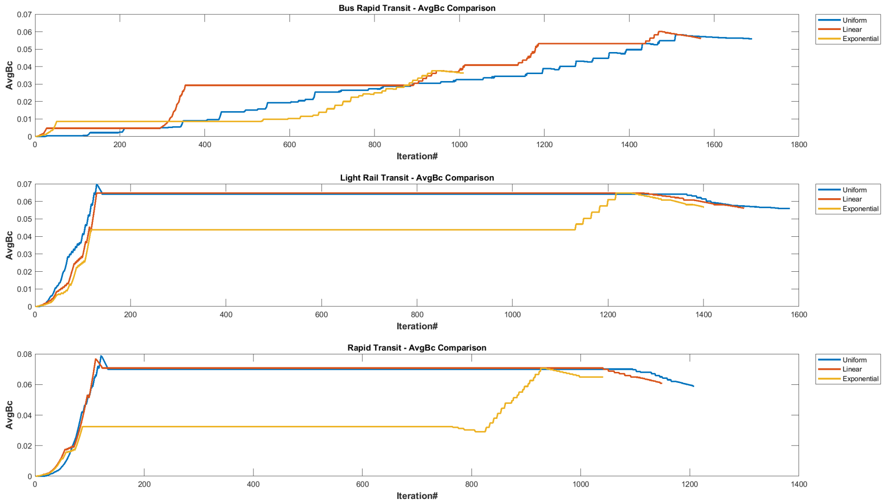


Figure 5.27: Comparison of Average Betweenness Centrality for various population distributions given a certain mode

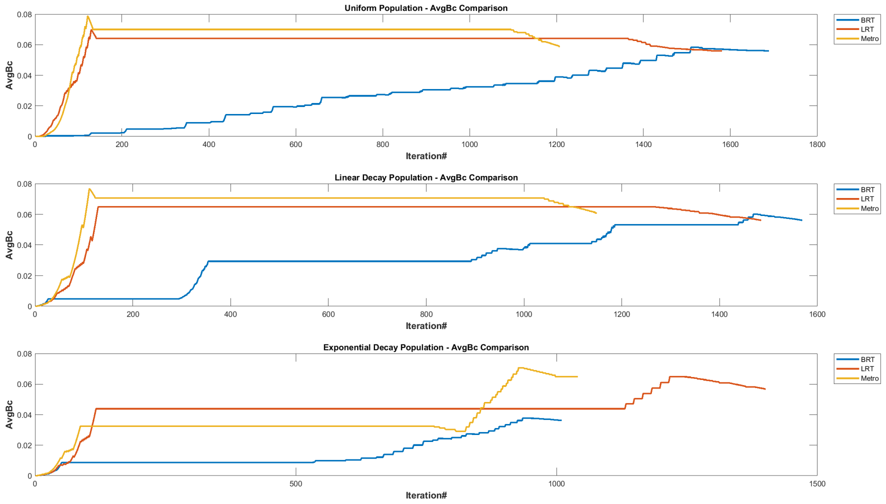


Figure 5.28: Comparison of Average Betweenness Centrality for various transit-modes given a population distribution

It can be observed that the BRT as earlier has a different evolution over time compared to the other modes. The exponential population distribution results in a more centrally focused network thus also in a lower average betweenness centrality and a distinct difference between all modes can be observed for that distribution. For most scenarios the process has already been determined as early growth phase, densification phase and a final phase of connecting ends. This indicator follows that behaviour but does show the (average) effects for nodes in the network.

Ringness

The Ringness is an indicator for the length along rings compared to total system length. This is a parameter suitable to define the shape of our netowrk within the polar grid structure.

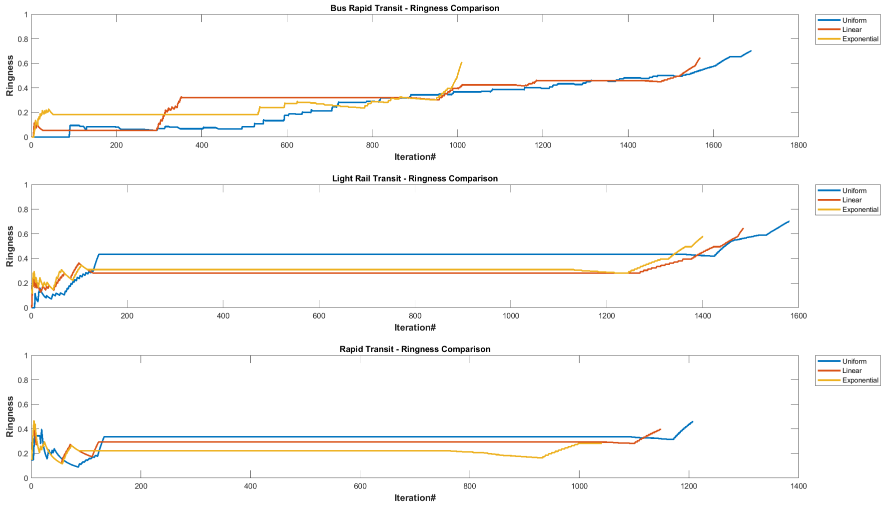


Figure 5.29: Comparison of Ringness for various population distributions given a certain mode

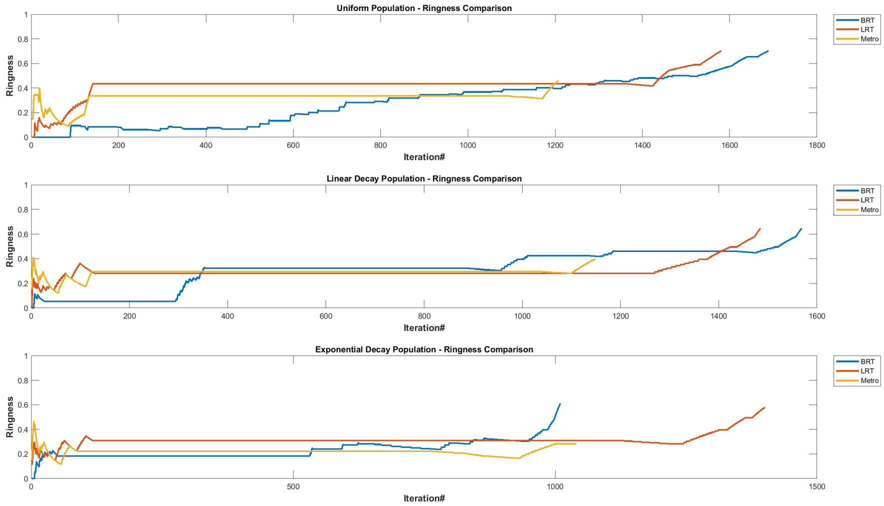


Figure 5.30: Comparison of Ringness for various transit-modes given a population distribution

Again the capacity increases do not affect the ringness indicator resulting in flat trajectories, but some interesting observations can be made. Rapid Transit initially adds ring-lines resulting in a peak of ringness before dipping again due to radial expansions. Almost all scenarios end with some ring-lines being added as the ringness increases towards the end of each respective plot. The evolutionary path can be observed in the plots and although final states might be similar, some variations can be observed along the evolution.

Average Detour Factor

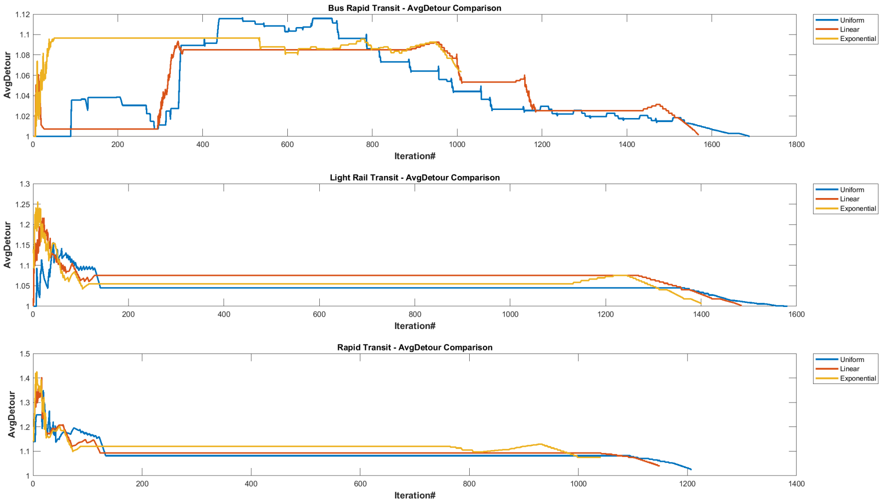


Figure 5.31: Comparison of Average Detour Factor for various population distributions given a certain mode

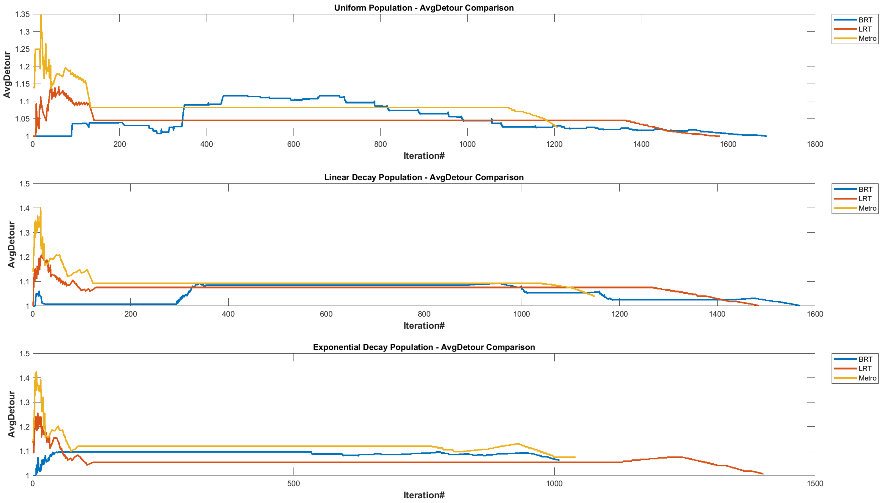


Figure 5.32: Comparison of Average Detour Factor for various transit-modes given a population distribution

As the name suggest the average detour factor is an another indicator related to the shortest path between two (connected) nodes. The network state shortest path is compared against the absolute shortest path. It can be observed that the average detour for BRT is volatile in behaviour and commonly has a high detour factor in the middle phase before decreasing towards the end (indicating the closing segments are constructed rather late), however when looking at the population distribution graphs for the same results this is less obvious. The LRT and Metro modes are similar in approach to building radials before rings and then in the final stage doing the final rings to create less detours. For the uniform mode the fluctuations are the largest, the exponential mode has a relatively small detour factor after about 10% of iterations.

5.2.3. Performance Indicators

Total System Length

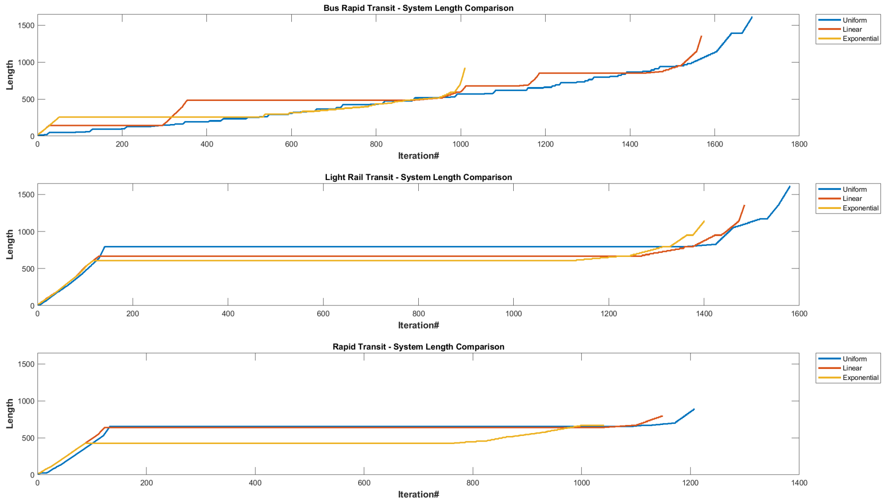


Figure 5.33: Comparison of System Length for various population distributions given a certain mode

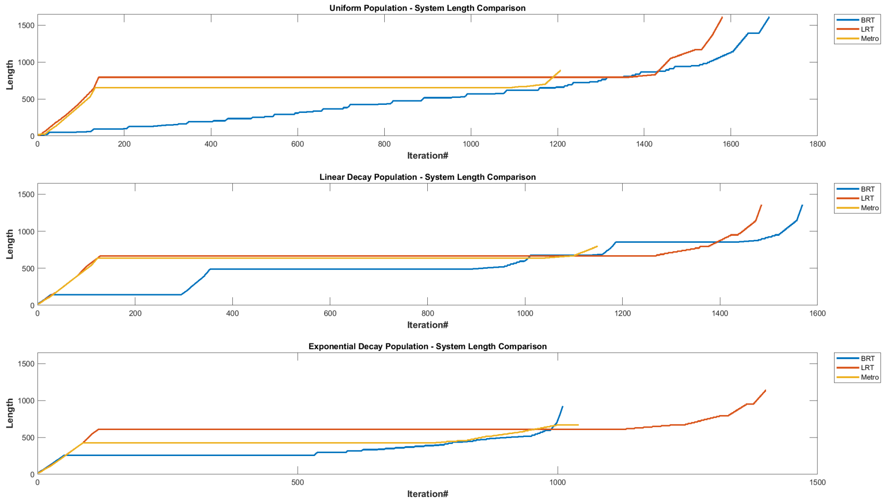


Figure 5.34: Comparison of System Length for various transit-modes given a population distribution

The Total System Length is not only interesting in the final state. Its evolution over time might help to provide insight in the choices. Capacity Additions do not contribute to an increase in the network length resulting in horizontal sections. The location of these horizontal sections provides some insight in the choices made. Similarly the final state lengths are not equal, thus the path to the final state can be observed for either a constant mode with varying population distributions (figure 5.33) or with varying modes given a population distribution (figure 5.34) resulting patterns are similar to earlier found observations.

Total Travel Time

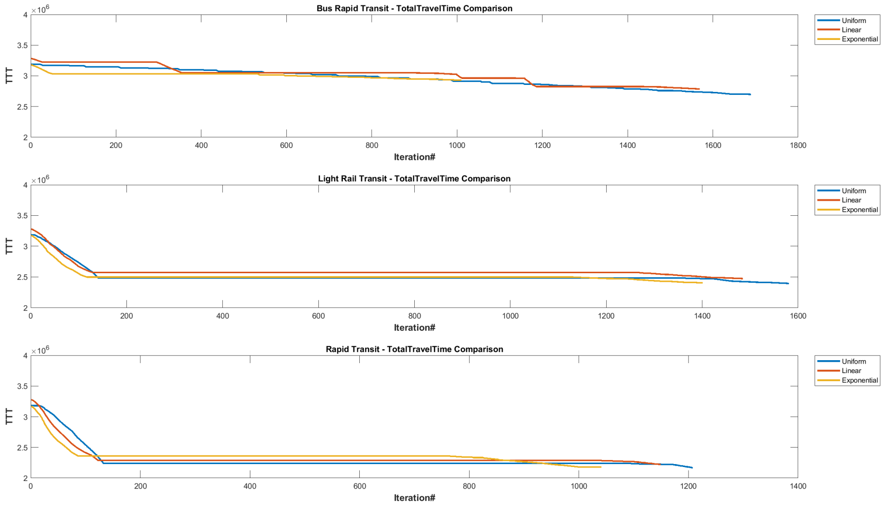


Figure 5.35: Comparison of Total Travel Time for various population distributions given a certain mode

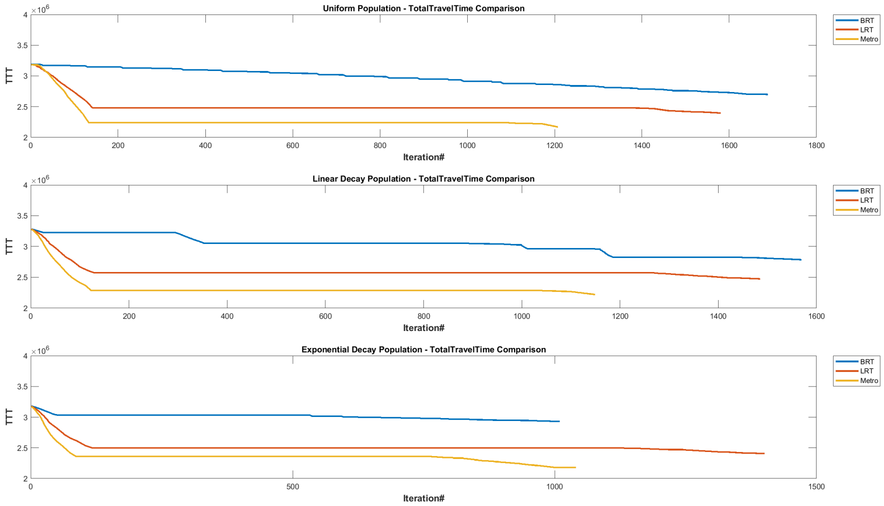


Figure 5.36: Comparison of Total Travel Time for various transit-modes given a population distribution

The Total Travel Time is computed by taking the travel time for each OD pair and takes into account the V_{mode} and V_{Alt} as well as the modal split from a logit model. Results can be seen in respective sub-figures. It is clear that when the difference between the operational speed and the alternative have past a certain level that the detour people are willing to take effectively means that the total system travel time will not drop after the initial expansion wave. This is most clear for Light Rail and Rapid Transit modes where the graphs get most of their total travel time reduction in the early phase. Do note that this does not take into account denied transit, this is just taking the modal-split and the expected travel time. hence it can be considered expected total travel time. In order to account for denied transit a more elaborate model is required that determines line operational usage and the location of denied travel. If all denied travel is taken as a percentage or if assignment is capacitated then results might differ. The portion of denied travel is constant over the investment choices thus will not impact the evolution of the network, as the investment choice module does contain a negative element for denied travel. The only effect implementing denied travel is a shift of (all) curves upwards.

Potential Ridership

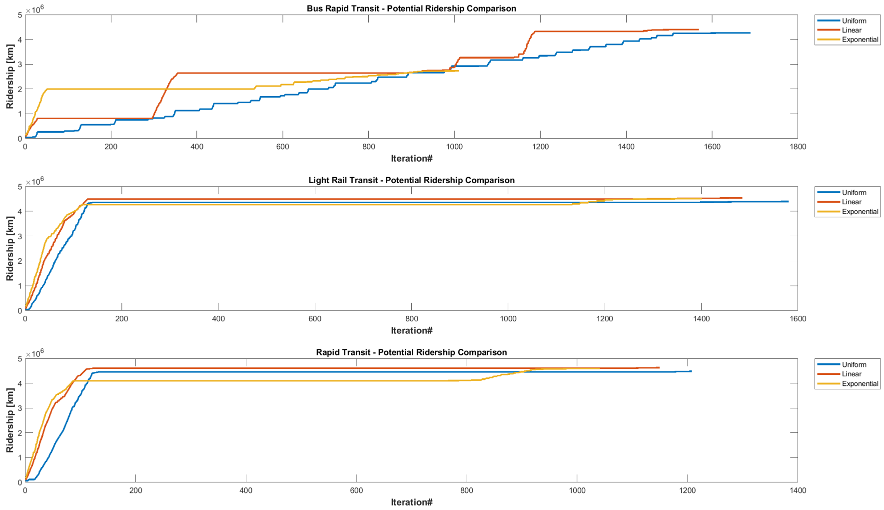


Figure 5.37: Comparison of Potential Ridership for various population distributions given a certain mode

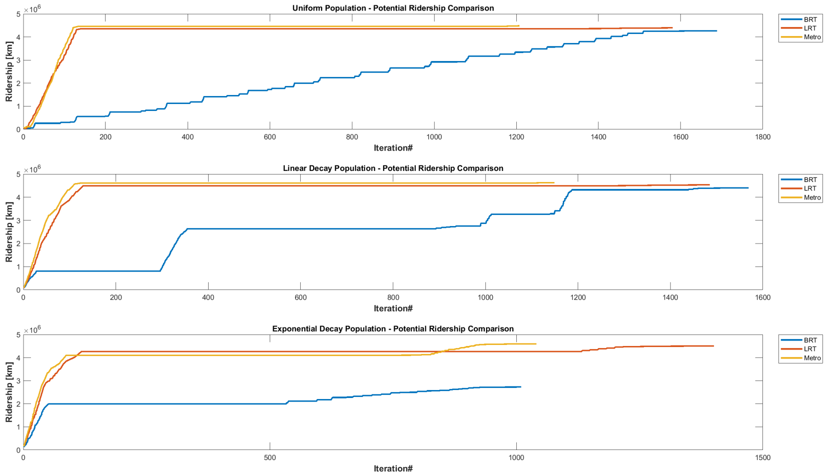


Figure 5.38: Comparison of Potential Ridership for various transit-modes given a population distribution

As mentioned in the Total Travel Time the modal split effectively only determines the fraction of people that would like to travel, and assignment is uncapacitated. This Potential ridership grows relatively quickly for the light-rail and rapid transit modes as their respective operational speeds mean that the logit model will quickly favour the travel time saving from the mode and thus more people switch from their alternative mode to the PT system developed. Again it can be seen that relatively large amount of the total potential ridership is achieved in early iterations as after that the capacity increase does not affect potential ridership (uncapacitated assignment).

Denied Ridership

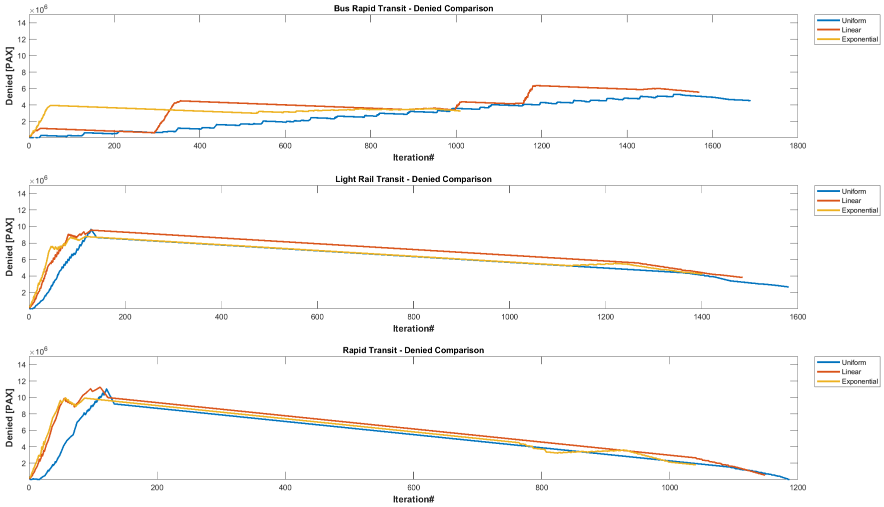


Figure 5.39: Comparison of Denied Ridership for various population distributions given a certain mode

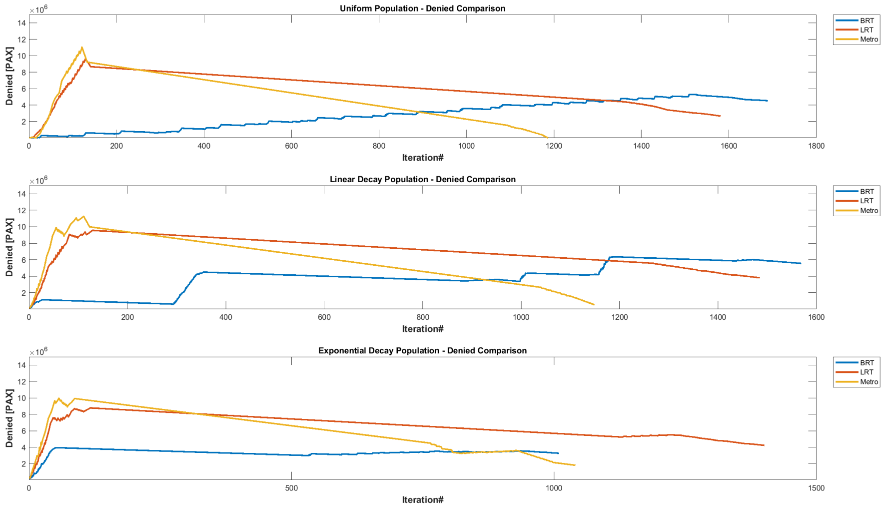


Figure 5.40: Comparison of Denied Ridership for various transit-modes given a population distribution

As a result of the initial "greedy" choices, picking up potential ridership the capacity is lacking behind and thus also quickly peaking in the early phase. later when capacity addition is done, we can see a steady decrease of denied ridership by capacity-unit/iteration until the final state is reached. This holds mostly for Light Rail Transit and Rapid Transit modes, as their greediness was largest. For BRT one can see an interesting pattern in the uniform case where the expansion is almost balanced, but the system capacity simply cannot cope with total demand on the service. This is partially because the OD demand is larger than maximum BRT capacity, resulting in a different pattern. For the various population distributions we can see that the results observed are less related to the distribution, and more strongly related to the mode effects.

Average Link Volume over Capacity Ratio

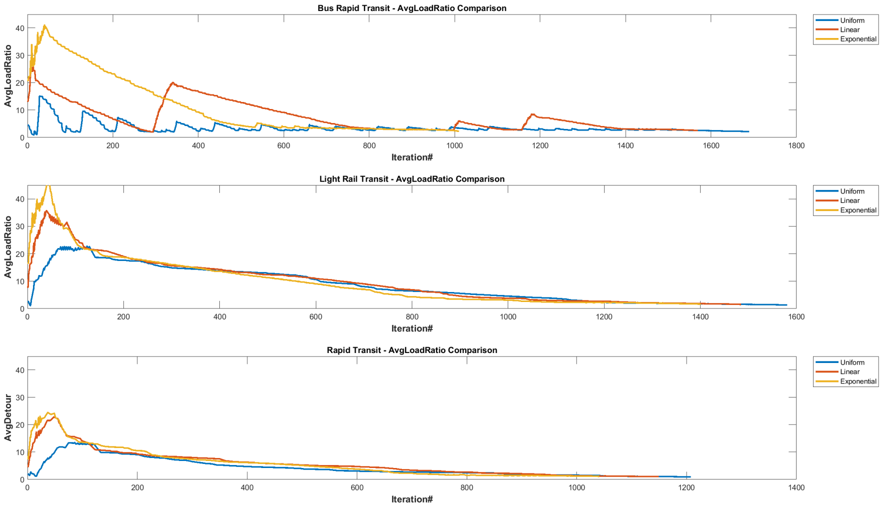


Figure 5.41: Comparison of Average Link Volume over Capacity (VoC) for various population distributions given a certain mode

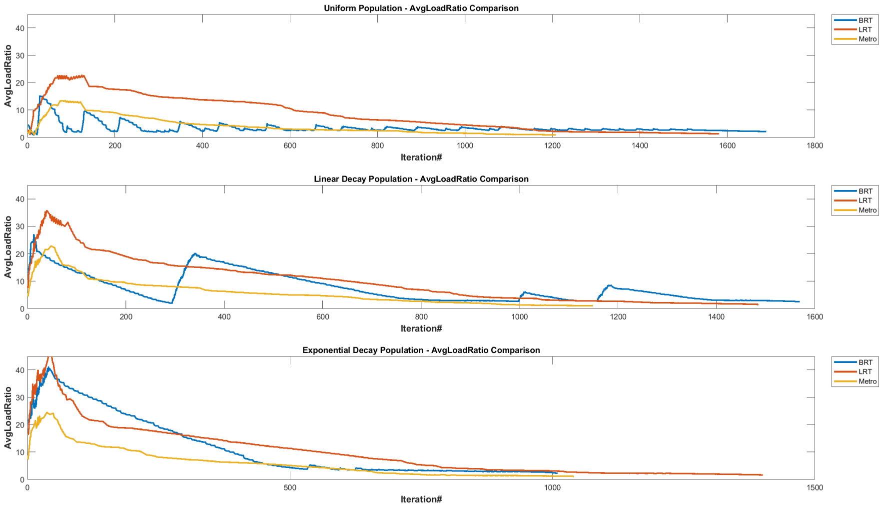


Figure 5.42: Comparison of Average Link Volume over Capacity (VoC) for various transit-modes given a population distribution

The average Volume over Capacity (VoC) is a ratio of actual usage versus link capacity at a given moment, and then averaged over all links for each iteration. This indicator can help to determine the level of overcrowding for this model and determine the impacts capacity expansion have on the average VoC ratio evolution, if any. Like previous indicators it can be observed that the BU scenario there is a slightly different pattern than for the other modes, but Light Rail and Rapid Transit clearly decrease their average VoC ratio in the capacity increase phase of the network growth. Similarly the effects are largest for the Light Rail Transit as can be seen in the lower sub-figure as this mode has a slightly smaller capacity when compared to rapid transit meaning there is a larger benefit of actually upgrading capacity.

5.3. Sensitivity analysis

As mentioned in 5.2 a sensitivity analysis might contribute to better understand the mechanisms of the model and the resulting network topology evolution. As such another set of experiments have been determined to investigate some of the relationships that could not be deducted from the initial experiments.

To investigate the effects of the operational speed for a given mode (V_{mode}) The population distribution is set to a uniform distribution, the modal costs are kept the same (BRT level) and the operational speed is varied: BRT with LRT speeds and BRT with Metro Speeds. This results in 2 additional scenarios to be run and gives one comparative scenario for analysis. The resulting data and figures are presented in Appendix C.

The effects of Investment Costs is approached in the same way. The operational speed is kept constant (45 kph) and the investment costs are varied (BRT-LRT-Metro level) given a uniform population distribution. This requires only one additional scenario and results in another comparative scenario for analysis. Again the results and figures can be found in Appendix C.

The effects of operational speeds are observed to be limited in terms of network development and size. The higher speeds do not result in a higher network connectivity, but this might also be related to the BRT mode being so cheap. It can be expected that the operational speed influences the final state network size in a positive way as higher operational speeds mean more potential travellers and thus a higher CBA ratio.

Similarly the effects of investment costs is a limitation of the network scale as longer edges with low benefits become affordable with a positive CBA ratio constraint resulting in a network that is more compact given the same population distribution and operational speed. Although this might be within expectations, the relationship is now also shown in the data-set generated by the experimental model. This relationship shows mostly in the LRT to Metro comparison, as for 45 kph the BRT and LRT networks are rather similar given the Uniform distribution. The Metro has a far larger step in investment costs, and therefore effects are clearly limiting the network growth for Metro. The ringness value for Metro is significantly lower meaning outer rings are not constructed, also shown in the final state graphs. other key indicators are following the expectation of the outer ring not being worth investing in, due to larger distance (investment cost). on of the interesting effects shown in the appendix is the evolution for the metrics in the Metro Network. The step-wise expansion/capacity investment seems similar to the original BU scenario, suggesting there is a specific relationship between cost and speed that results in this pattern.

5.4. Evolution Visualisation

In order to visualise the evolution of the various scenarios some intermediate stages are plotted in the following sub-graphs. These are fixed points for comparison after 100 iterations, 250,500,750,1000 and final stage respectively. These graphs help to visualise the network evolution. The networks are shown by link loads in each time-step showing potential capacity hot-spots in the network for each stage. These intermediate steps help to visualise the network decisions made and the resulting networks at various time steps can be compared graphically.

5.4.1. Bus Rapid Transit evolution

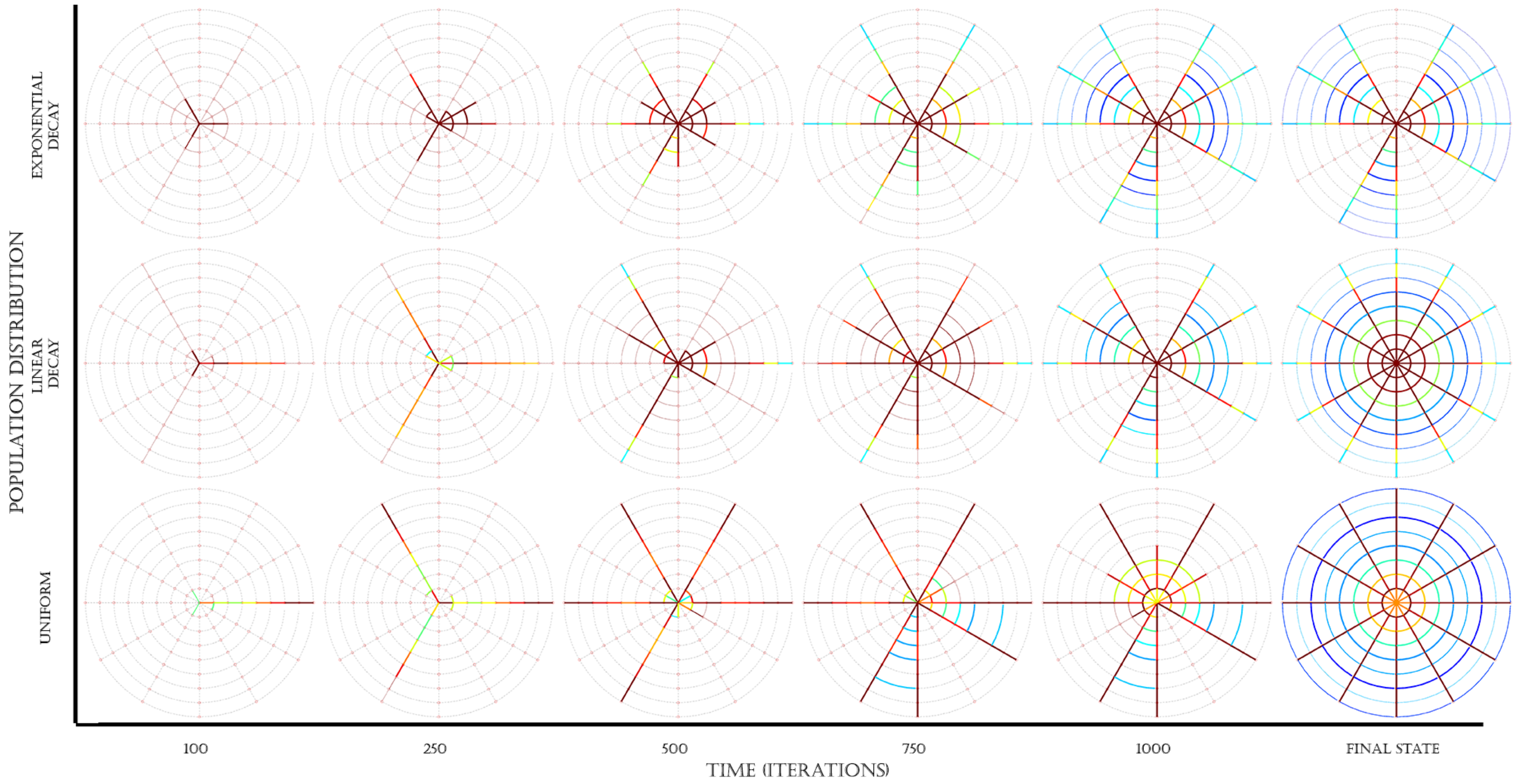


Figure 5.43: BRT - Network State at given Time moments

5.4.2. Light Rail Transit evolution

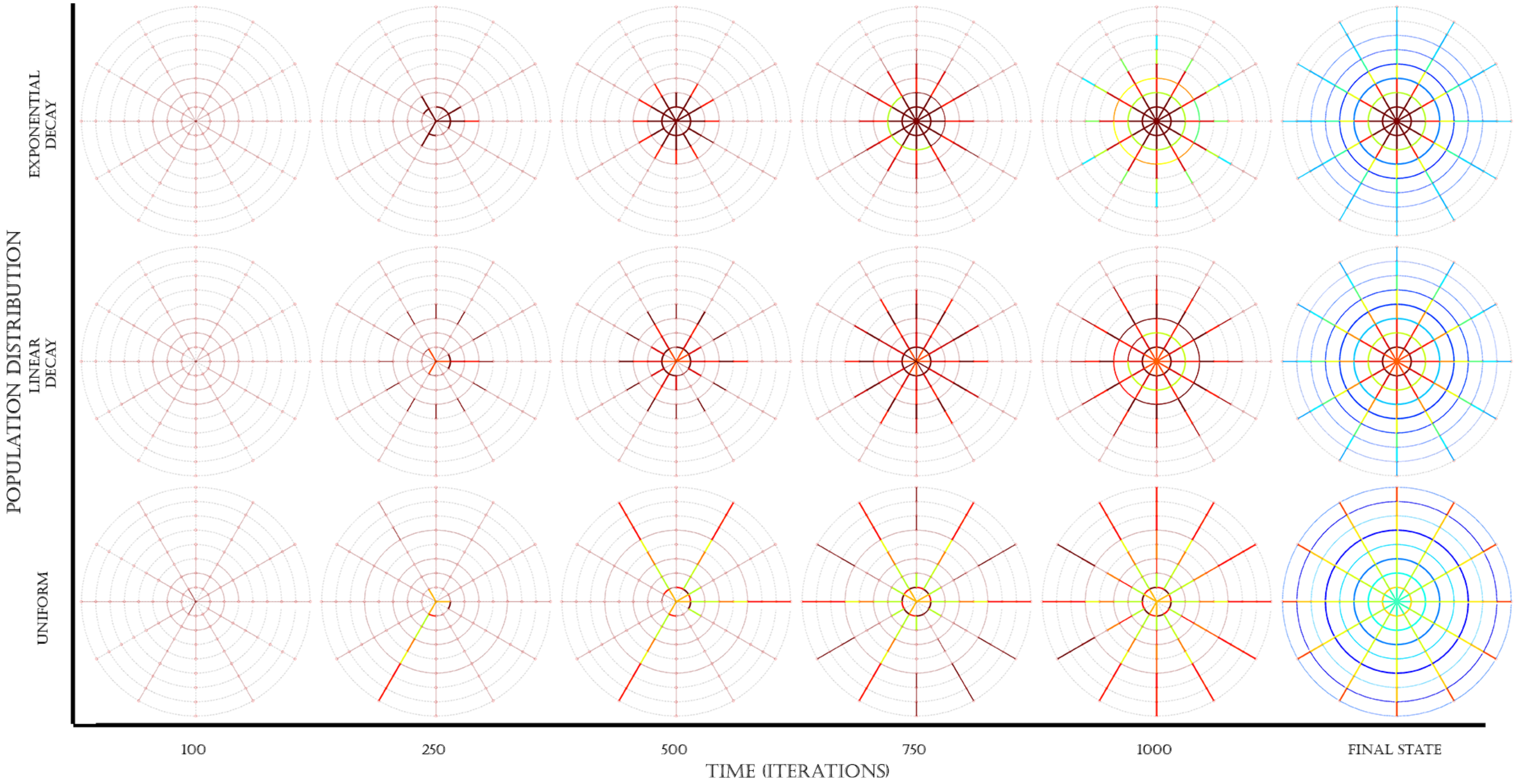


Figure 5.44: LRT - Network State at given Time moments

5.4.3. Rapid Transit evolution

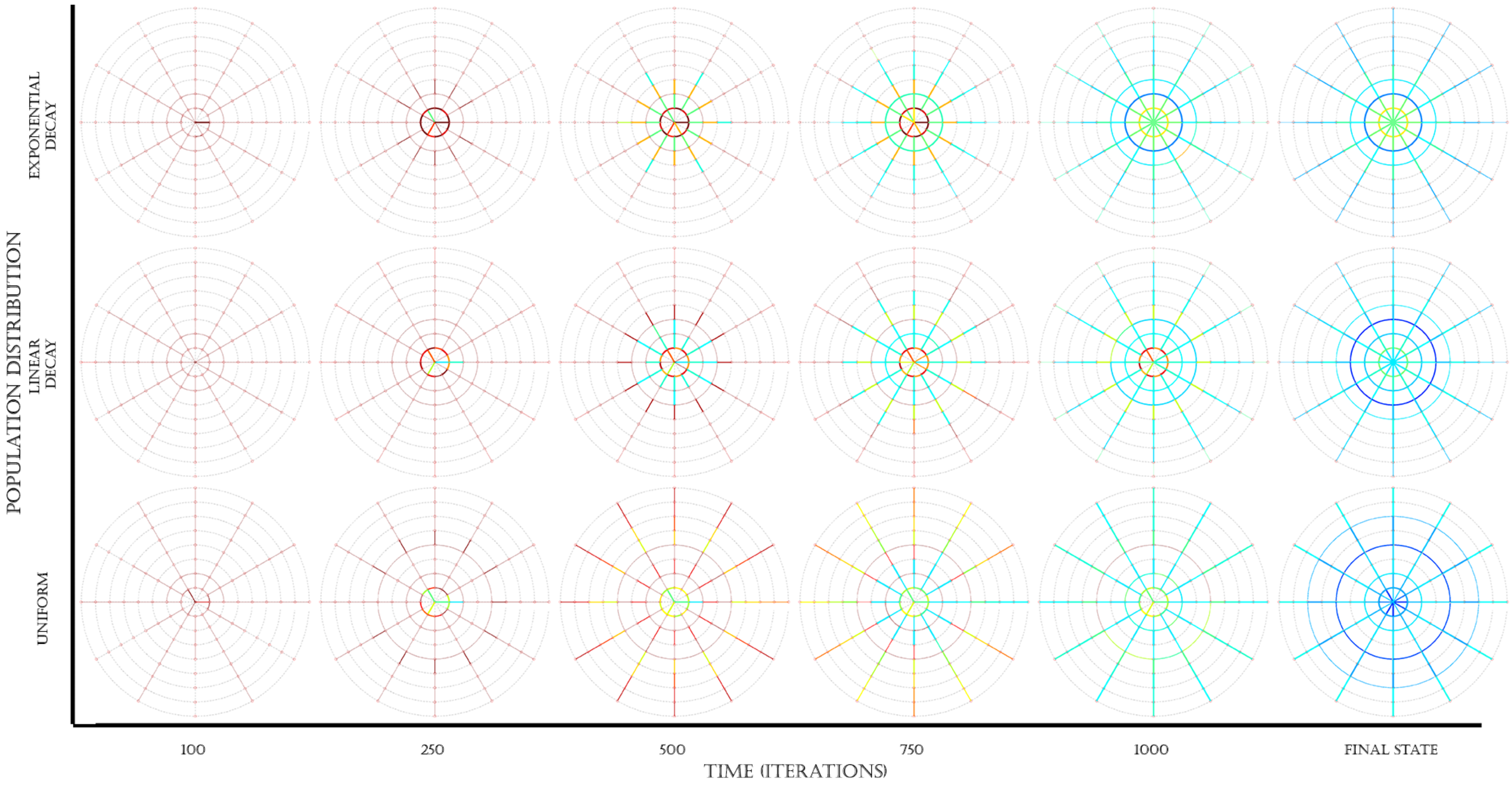


Figure 5.45: Metro - Network State at given Time moments

5.5. Model applications and potential contributions

The presented model can be of help by providing a ring-radial growth model for transport networks comparing the effects a PT network has given an assumed alternative mode of transport. It implements the Cost Benefit Analysis, a common tool in investment evaluation and discounts for the time-planning horizon of 30 years, again a common practise in transport investments. The model is helpful because it will show the impact a decision has in network metrics and the detailed elements of the Cost Benefit Analysis.

The simulation model relied on basic policy decision rules of Cost Benefit Analysis and requires only little inputs from policymakers to get an understanding of the network evolution process. Output can be visually presented from any state (including a network that a policy maker has to make an investment decision for) and compute any time-horizon until there is no improvement possible. For each step the network metrics can be computed and compared for improvements. The Cost Benefit Analysis helps to show the trade-off a investor might face during future transport network expansion evaluation. Either invest in a higher capacity for an existing link or constructing a new link connecting additional population to the network. The model presented helps to quantify the impacts of both investment decisions and shows the resulting CBA allowing for a informed decision by the policy makers.

The model was designed to provide insight in underlying mechanisms of transport for decision makers in public transport network development. Either the network operator, governmental bodies or any other party that has an interest in developing a public transport network in a metropolitan setting. It can assist in selecting a specific mode for a given link or set of links. If the model predicts that ridership is very high and the denied ridership is high due to the mode not having enough capacity in peak hour, a mode with higher capacity can be selected based on the findings of this model (assuming the investment is feasible in the model with this higher capacity/cost). Finally the model can be used to determine the equilibrium state of a fully matured (no positive CBA investments possible) network given currently available information. This might help planners and network operators to see how much investment is still possible within the network limitations provided.

Real World Structures

The figure on the page below shows various metro networks around the world. Networks in the figure range from non-complex (bottom) to very complex (top). Although real world cities have different shapes due to geographical, social and/or economical factors, some patterns in these networks can be observed. Most of these networks have a strong (urban) core where multiple lines intersect. The lines can then be considered radials like in our evolutionary model. In the more complex networks at the top some connections between radials can be observed. This corresponds to ring-lines being connected in our model. Given a likely exponential decaying function for population in cities these patterns match the patterns can can be observed in the evolutionary model for metro in a exponential decaying network.The strong radial tendency towards the edges and one or more connecting rings to shorten the travel-times for existing pairs of stations/nodes seems to be the driving factor. Where like in our model a Cost Benefit Analysis likely prevents the final ring-lines on the outskirts to be build. For more complex networks we can observe that some of these networks have multiple cores or seem to have more places where some of the radials intersect. (mostly top row) However the second and third line networks have strong resemblance to stages within the Rapid Transit Evolution as presented in figure 5.45 and might indicate that observed patterns in the evolution occur in real metropolitan areas .

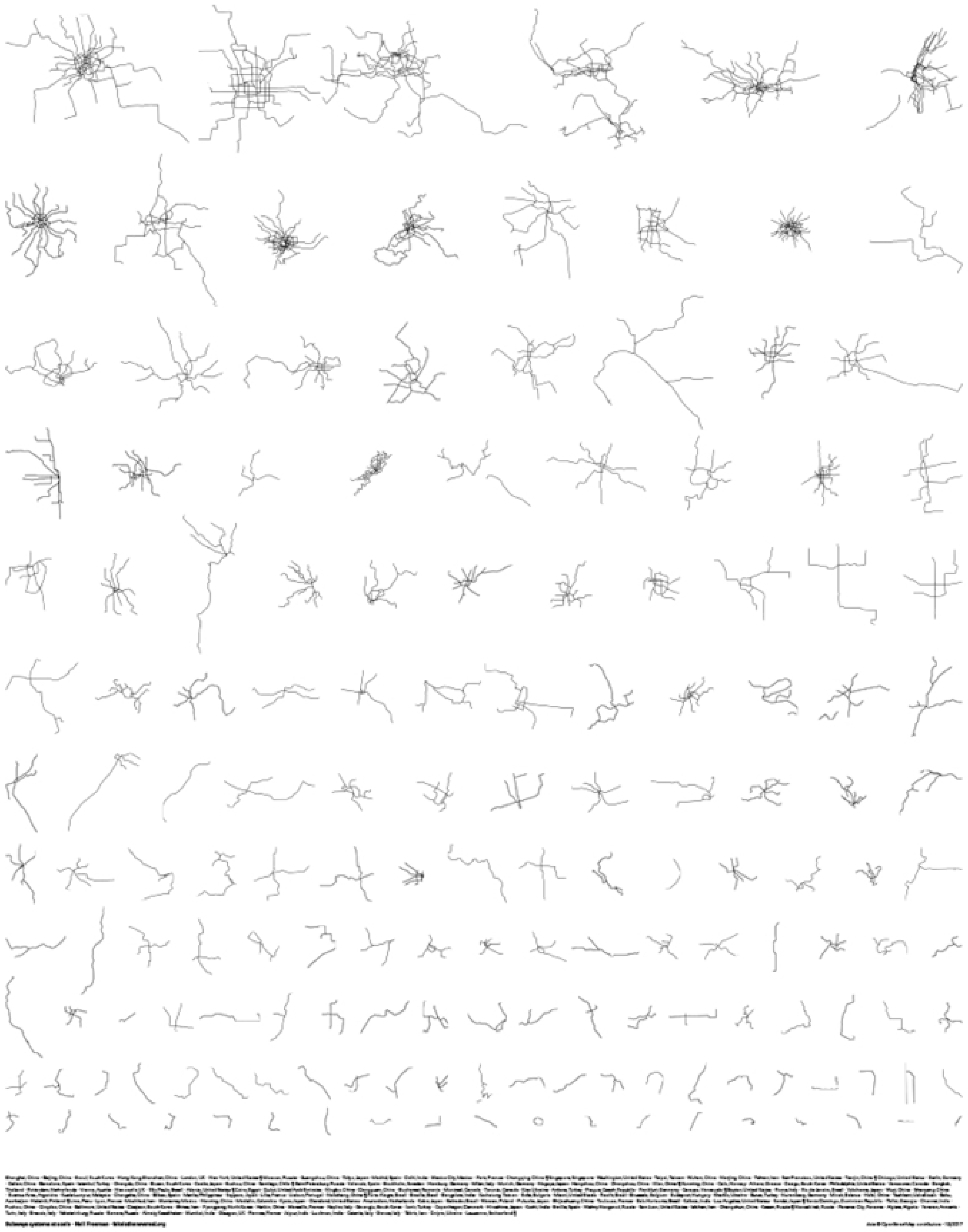


Figure 5.46: Various networks in the world Image by Neil Freeman

6

Conclusion & Recommendations

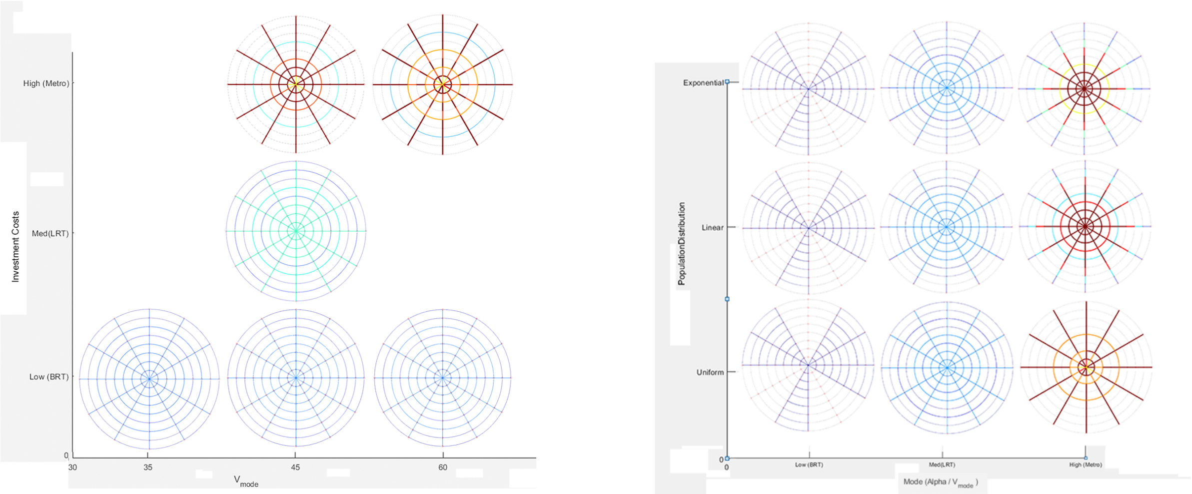
This chapter concludes the reported research. In section 6.1, the research objective is restated and conclusions are drawn based on the previous chapters of this research. In section 6.2, the conclusions are discussed in the light of the assumptions that were made and concluded with a number of recommendations for further research.

6.1. Key findings

The results from sections 5.1 and 5.2 support the suggestion of a relationship between the population distribution and the final topological evolution of a network. The actual network metrics do not account for capacity and thus do not really show these effects as clearly as expected. As mentioned in section 5.2 the scenarios affected by a decaying population show limited connections in the outer periphery and a limitation of capacity for links that are constructed in these peripheral areas. This holds for all scenarios regardless of the investment cost set by transport mode parameters, meaning this effect is related to the population distribution. When looking at the key performance indicators one can see that scenarios with a linear decaying population distribution has a relatively high Average Betweenness Centrality and a detour factor that is on average higher than the uniform case. The total travel time is average placed right between the relatively high total travel time of uniform distributions and lower total travel time for exponential distributions. This might seem counter-intuitive but as more of the population is concentrated in the inner centre the relative distances between nodes is shorter and the travel time in relation to the trip distribution by the gravity model is lower, a population distribution driven effect.

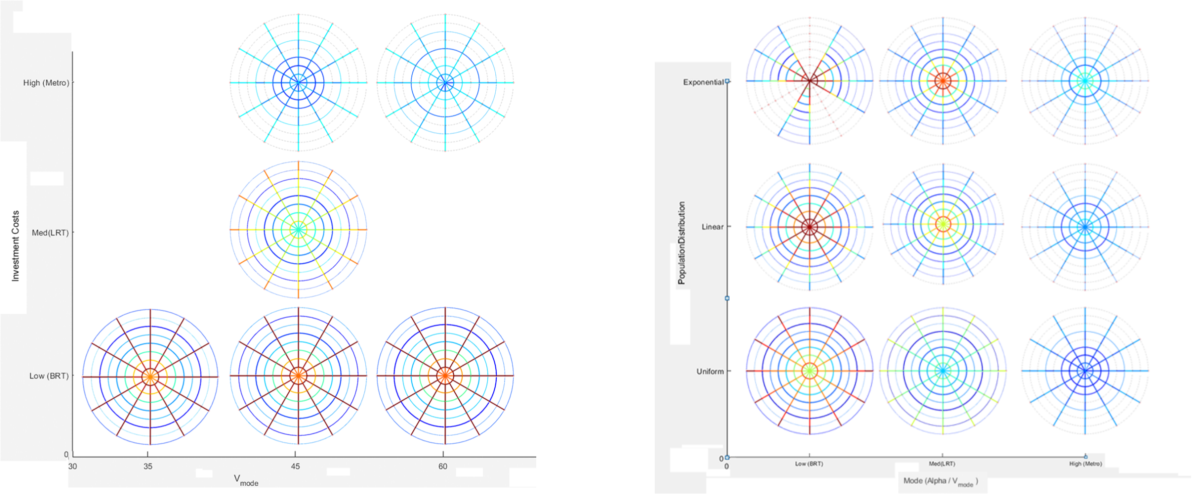
When looking into the effects that public transport modes have on their respective network evolution and metrics of the evolution, sections 5.1 and 5.2 clearly show the influences that mode parameters have on the final results for some of the key performance indicators. The mode affects the network evolution in two distinct ways. First of all the mode speed has an influence on the potential travel time a link has, and thus affects the choice model (logit). Higher speeds are beneficial up to a critical value, after that the whole potential change has been realised. The exact figure for this switching point is determined by the logit choice model and the selected alternative mode characteristics, and is estimated to be within 10-20 kph difference range given the results of BRT, LRT and Metro from this model. This mechanism means that higher speeds (LRT/Metro) will likely have more of a complete network due to the potential benefits for users being higher, affecting the Cost-Benefit Analysis positively resulting in a higher ratio, thus a larger chance a link is invested in. The other investment mechanism modes affect is the costs of construction/adding capacity. These costs are a modal parameter defined as $Alpha$ and their effect is on the other side of the cost benefit analysis namely the cost side. The higher the costs/km the higher the total investment costs and the more likely it is that a link is not constructed. This is particularly true for outer periphery of a monocentric ring-radial structure, where the outer edges cover a relatively large distance between two equal-radii nodes. Given similar demand levels and trip likelihood (not true due to gravity-model, but assumption) the higher costs would imply a lower CBA ratio, meaning the likelihood of construction is decreased. Radials are uniformly defined for our experiments, thus the likeliness of a ring is decreased whereas the likeliness of a radial is relatively constant assuming the trip demand is equal. In order to investigate the relations posed in this paragraph a sensitivity analysis has been performed.

The results described in previous paragraphs are further supported by the results in the sensitivity analysis of section 5.3. Results from the sensitivity analysis show that for same mode but different operational speeds the network is more developed. It also indicates there is a threshold as (even) higher operational speeds did not result in more potential ridership being attracted. This is caused by the logit model. The results from the second sensitivity analysis scenario indicate the relationship between investment cost and the final network state. Same operational speeds and population distribution result in different network evolutionary profiles. This also helps explaining some of the results we have seen in the earlier experiments. Given the parameters provided, the BRT mode has an operational speed that is not large enough to obtain the full benefits of travellers in the logit choice module. Metro investment costs are so large that outer rings cannot be constructed, and the LRT mode apparently has parameters such that a full graph can be considered viable investments as long as the population distribution supports is favourable and relative distances are as selected in this research. The graphs below represent the shapes of the network based on the parameters plotted in a plane. This provides some insight as to the effects parameters have on the network shape and where intermediate values might be located. Sub-figures 6.1a and 6.1b show transported passengers, 6.1c and 6.1d show passenger loads for the various scenarios. Scale for these graphs is same as in previous results.



(a) Ridership for various scenarios (Cost/Speed)

(b) Ridership for various scenarios (Mode/Population)



(c) Link Loads for various scenarios (Cost/Speed)

(d) Link Loads for various scenarios (Mode/Population

Goal of this research is to determine the effects various demand distributions and operational cost functions have on the evolution and shape of a monocentric metropolitan transport network evolution using network indicators. In order to do this an investment model network analysis framework has been developed that systematically evaluated the impact the population distribution and transport mode inputs have on the network topology, evolution and growth of a metropolitan transport network.

6.1.1. What effects do various demand distributions and operational cost functions have on the evolution and shape of a monocentric metropolitan transport network when evaluating them using network indicators.

The answer to the main research question has to be given in two separate steps. The first partial answer considers the effects demand distributions have on the evolution of a monocentric metropolitan transport network, this will be discussed in section 6.1.2. The second partial answer considers the effects various modal properties (like operational costs) have on the evolution of a public transport network, which is discussed in section 6.1.3. Finally the actual method of evaluation of these effects is discussed in section 6.1.4.

6.1.2. How do various demand distributions impact the growth of monocentric metropolitan transport networks?

As can be seen in 5.2 the influence of population demand distributions on the network topology are a shift of capacity and links in the network to the inner core in order to match demand levels. Cities that have a strong central core will thus in this model have a network that has strong connections within that core and peripheral connections will be valued less or not invested in based on the demand level and the cost factors for the PT network. The effect population distributions have on the network development is further strengthened by the Gravity Model that has a travel impedance based on distance and the mathematical relationship dictating that distances between nodes along the rings are always longer than intermediate distances along a radial. The resulting population distribution therefor might be focused on shorter distances and nearby nodes.

6.1.3. What influence do operational cost functions have on the growth of monocentric metropolitan transport networks?

As can be seen in 5.2 the influence of operational costs on the network topology have been determined by defining transport modes with their typical characteristics found in literature. The resulting network topologies and evolution's have been analysed and it is clear that there is a strong relationship between the final topology and the modal characteristics of operational speed and infrastructure costs. These relationships have been investigated further in the sensitivity analysis for specific mode parameters (section 5.3). The outcomes from the sensitivity analysis support the earlier assumption that the cost effects mostly affect outer ring-lines whereas the selected operational speed mostly affects the network potential benefits and affects both rings and radials equally. The generalised results from section 6.1 indicate this as well. Lower speeds mean that the mode is not attractive enough in the logit choice model, resulting in fewer travellers and thus might result in a network where the costs outweigh the benefits. Whenever the investment costs are set too high, the network benefits cannot be obtained because the CBA ratio is affected to a level where investment in a link is not worth it for the current mode.

6.1.4. What indicators can be used to describe urban public transport network evolution?

Within this research various network metrics have been discussed. In order to describe the evolution of a network these metrics have to be monitored over time and the resulting relative changes indicate a trend in the metrics and can help identifying specific growth mechanisms or investment choices. The final state indicators themselves are not interesting, but only are useful in combination with each-other. There is no single network or performance indicator that can describe the process or network state at a single time moment, however the combination of selected indicators provides some insight in the underlying mechanisms of network evolution in a ring-radial grid. The magnitude of some selected KPI are directly related to network grid definitions, however it is expected that the investment choices drive all of the selected KPI. Some of the indicators have overlapped thus do not really add additional information. An example of this are the β and γ indicators that both increase for same link-addition processes. Partially related to the planar grid-setup where the connections are limited.

6.2. Limitations of this research and recommendations for future research

The experiments in this research, as mentioned before, are an abstraction of the real world. In order to obtain a model that would run in a timely manner assumptions and simplifications were necessary, and the results must be viewed given these assumptions and simplifications. These assumptions and simplifications do mean that a number of recommendations for further research can be made. In order to structure these they are presented as the list of limitations of this research first and then a discussion of these limitations and how future research can contribute are mentioned. For this we separate two categories. Limitations of the model that can be solved by improving the quality of existing modules, inputs or parameters used, and future modules that can be added in order to overcome some simplifications or assumptions.

First we will discuss limitations of this model that can be improved by refining the model elements or its inputs chosen in this research. For that we will look at the core assumptions of this category:

1. Population located in Nodes & Constant radial Population function
    - In this research the latent demand was fixed and located in nodes. This results in a neglect of access egress times for public transport a simplification that could result in a longer total travel time making the PT networks relatively more attractive in our model. Zonal distribution of population and a more detailed OD coordinate pair would be preferred but is also harder to model within the numerical limitations.
2. Simple Cost Function
    - Costs have been taken into account in a very simplistic way.The simplification applied here is a generalisation of all cost elements into a single cost per kilometre for constructing a link or upgrading it by a single capacity entity. Whilst a singular value for costs can be assumed as average value, it is recommended to investigate other cost structures, such as a functional split of costs or complex cost functions involving infrastructure construction, maintenance costs, vehicle costs, personnel costs.
3. Constant Alternative Mode
    - The "Alternative Mode" was included to have a finite improvement when adding a new node to the network. To reproduce new ridership being attracted some "alternative" had to be considered. The impact an alternative mode has on the network growth has not been investigated, as for comparative purposes the Alternative mode was kept constant. Future research might wish to see the impacts alternative modes have on the network evolution and can vary the parameters for the alternative mode as they see fit.
4. Polar Grid Size
    - The influence of the scale of the polar grid is not researched and not part of the scope. A grid with parameters suitable of a metropolitan area has used within this research. In order to investigate the impact the grid has on potential growth scenarios, additional research can be performed by changing the input parameters and comparing results with this research.

The second category of limitations for the model consists of assumptions that can only be improved by adding new features to the model, essentially adding modules and increasing its complexity. Core assumptions for this category include the following:

1. Line operations are not considered
    - Line operations, stop spacing and frequencies have been ignored to reduce complexity of the solution-space. This means that a more detailed computation of generalised passenger costs like Saidi was not possible, but the overall network structures can be realised regardless of exact line configurations, as those operations where not part of the scope. Furthermore the Network estimation consisted of singular investment options that consisted of individual line segments between two nodes. Future research could implement investment packages or line operations by considering multiple investment steps as a single line investment or define line operations within a ring-radial network.

2. Land development is not considered
    - The interaction between land development and the development of PT networks was not included in this research. More specific there was no growth of population or demand over time, meaning that the total demand was constant and only a a shift in the modal split between the mode and the predefined alternative was possible.
3. Uncapacitated assignment
    - The assignment of passengers did not account for the capacity a PT link has. This obviously means the PT line is more attractive than it will be in reality and results will be affected. The choice for uncapacitated assignment was made for a reduction in computational efforts and the fact that adding capacitated assignment could make solutions path dependant an undesired effects for the current purpose. A recommendation for future research would be to implement a capacitated assignment that will take the capacity of a PT mode into effect when distributing the passengers over the modes and investigate the impacts this might have on evolution. This implementation will sacrifice run-time in favour of accuracy and thus would require far more time or computational effort.
4. Uni-modal, no multimodal trips
    - The chosen model network is uni-modal, meaning there is no possibility to do multimodal travel in this model. Either the alternative mode was selected for the whole trip, or the PT network, no combinations allowed. Future research could investigate different network levels being developed simultaneously and the impact a multimodal trip could have on the development of each network level, at the cost of computational efforts.
5. Cost Benefit Analysis variations
    - The Cost Benefit Analysis is a trusted tool for investment decision making within public transport. What factors to include in a CBA and what elements are included are part of the decision-making process and therefore an additional module could be implemented that will take in additional effects caused by land use, pollution reduction and other indirect effects.

A

Model Functions

For a full copy of the model please refer to the open-source copy of this code on GitHub:
https://github.com/ZxBiohazardZx/Radial-Network-Evolution-Model

A.1. Global Inputs & Definitions

The model will require user inputs on the following global parameters related to the network grid and some inputs are simply there to ease up calculations. The model requires the user to decide on R,ϕ and S to determine the number of nodes. The user will also need to provide a fully connected planar graph adjacency matrix

R Maximum radius of the Network

ϕ_{deg} Angle between two radials of the network. Used to determine segments, given in degrees. if this is altered the full Network Adjacency Matrix will need to be altered as well.

S number of ring lines the network will house. Default is 4 rings, if this is altered the full Network Adjacency Matrix will need to be altered as well.

ϕ Angle in radians (converted ϕ_{deg})

$v_{Mode}; \alpha; Cap$ Mode parameters will influence operational speed V_{mode} and operational cost α as well as Capacity and MaxCapacity values.

PopDist The population distribution is a type selector (1=uniform, 2=linear decay, 3=exponential decay) setting the constraints for the gravity model and population distribution.

Nodes Structure containing all locations of grid points (r,phi,population).

Links Structure containing all existing Links (Node1,Node2,Length,Capacity,MaxSpeed).

MaxPlanarGraph Determined the max planar graph and all relevant properties (network distance matrix, adjacency matrix)

A.2. Functions

The model relies on the following MATLAB functions. A summary of each function is provided as well as the input and outputs it requires. The functions are provided in the order they are called in.

GenerateGrid .

Inputs: R,ϕ,S,PopDist <60km,30deg,4 rings,1>
Method: Defines the grid-points (Nodes) for the network. Using the input parameters the function generates a polar grid and outputs the coordinates of the nodes in a structure of Nodes (r,phi), node coordinates are $\in ([0-R],[0-2PI])$. the population distribution parameter means the nodes all get a given population based on the distribution type (1,2 or 3)
Output: Nodes (Polar Grid for the network)

Dijkstra .

Inputs: Adjacency Matrix, Distance or Cost Matrix
Method: Given vertices and edge connections for a graph (represented by either adjacency matrix or edge list) and edge costs that are greater than zero, this function computes the shortest path from one or more starting nodes to one or more termination nodes.
Output: [costs] is an LxM matrix of minimum cost values for the minimal paths [paths] is an LxM cell array containing the shortest path arrays

NetworkMatrix .

Inputs: Nodes, Links
Method: Determine the Adjacency Matrix and Cost Matrix for given Nodes/Links
Output: [NetMat] is an NodeIxNodeJ matrix of LinkLength values if and only if a link connects the nodes(i,j) [AdjMat] is an NodeIxNodeJ Logical Matrix with a 1 if and only if a link connects the nodes(i,j)

GenerateCandidates .

Inputs: Network Adjacency Matrix, Current Adjacency Matrix.
Method: Determine all Candidates that are possible given current Adj matrix compared against the full planar graph. Construct a CandidateList by taking non-zero elements in the CandidateMatrix.
Output: List of Candidates for the current iteration. Listed in a structure.

calcCanProp .

Inputs: Candidate, Nodes, Links
Method: For each Candidate this function will call multiple functions to determine the candidate's properties for evaluation. This is a multi-step approach: First Build the Candidate Link (FunctionCall) and store the new link-list in Candidate.Links, then determine the new NetworkMatrix (FunctionCall),Third apply Dijkstas shortest path algorithm to determine the network distances between each OD pair and the path-steps used (FunctionCall) and finally call the scoring function module to compute the Score (FunctionCall).
Output: properties of the candidate stored in the structure:
Candidate.Links
Candidate.NetMat
Candidate.Costs
Candidate.Score
Candidate.path

calcLinkLength .

Inputs: Nodes, Link
Method: calculate the length of the input link and output its measured length.
Output: Length of the Link.

BuildLink .

Inputs: StartNode,EndNode,Nodes,Links
Method: Add the selected link to the list of Links.
Output: Updated Links List

CalcScore .

Inputs: Candidate for evaluation, Current Network TravelTimes, ODMatrix,MinimumTravelTime
Method: Call OperatorCosts and UserBenefit Functions and then compare both for normative scoring
Output: Score for given Candidate

OperatorCosts .

Inputs: Candidate
Method: Multiply new link length with a construction cost factor
Output: Construction or Operator Costs for the given candidate

UserBenefits .

Inputs: Candidate for evaluation, Current Network TravelTimes, ODMatrix,MinimumTravelTime
Method: Determine fraction1: Current Shortest Path / Absolute Shortest Path; Determine fraction 2: Candidate Shortest Path / Absolute Shortest Path; Determine the difference fraction2 – fraction1 Multiply δ_F with the OD Matrix and summate to find the net benefit.
Output: User Benefits for the given candidate

logitUtility .

Inputs: LBeta,TTAlt,TTPTCur
Method: calculate the model split for the given traveltime matrices and beta (1)
Output: Share of Cars, Share of PT

LinkLoad .

Inputs: Links,Demand,Paths
Method: calculate the loads for a given edge (link)
Output: Links updated with loads

calcIndicators .

Inputs: Nodes,Links
Method: Using predefined Network Key Performance Indicator
Output: Graphical representation of the polar grid used to construct the network.
Beta = (numel(Links) / numel(Nodes))
Gamma = NNZ(AdjMat) / NNZ(NetAdjMat)
TTL = sum(sum(links.length))
Diameter = max(max(costs))
$\overline{k}$ = function-call (graph build in function)

Visualisation Module .

In order to visualise the network a Draw Function has been designed that will plot the state of the network inputs. The draw-function consists of two elements: DrawGrid that will generate the plot area, determine node locations and show all potential links in a grey-dashed line, and DrawNetwork which draws all the network links added by our model.

DrawGrid .

Inputs: R,phi,S
Method: using the plot function, generate the plot area, determine node locations and show all potential links in a grey-dashed line
Output: Graphical representation of the polar grid used to construct the network.

DrawNetwork .

Inputs: Nodes,Links, CapValue, MaxCapacity
Method: using the plot function, generates a visualisation of the network links build by drawing each of the link elements on the plot from DrawGrid.
Output: Graphical representation of the network.

ComparePop .

Inputs: History of all scenarios
Method: given all the stored data, plot comparison plot for various populations
Output: Graphical representation of the network indicators (comparative).

CompareMode .

Inputs: History of all scenarios
Method: given all the stored data, plot comparison plot for various modes
Output: Graphical representation of the network indicators (comparative).

B

Verification

B.1. Unit Tests

The modules that are present in the model are all different and thus require different testing methods. Each module/unit is tested in its own way and the paragraphs below state the method of testing and the role a module has.

MainFile

Description: The MainFile is the driver file calling all other functions. It also contains the initial variables and the associated workspace will be used to store output for our model.
Tests: The Test for this file consists of a check that the file runs without any errors and that the variables defined are created in the MATLAB workspace. This also includes some smaller subfunctions to setup the standard network, the node locations etc.
Status: Verified & working as intended.

Dijkstra

Description: Dijkstra's Algorithm is used to generate the shortest path lengths and path elements that indicate the shortest path. This is a very common algorithm and thus the implementation from Joseph Kirk has been used.
Tests: The algorithm has been tested using various small subsets where paths are known. All paths and lengths have been verified.
Status: Verified & working as intended.

Gravity Model

Description: The gravity module is a function that generates the Latent Demand (OD-matrix). This module implements a doubly constrained gravity model using the distance matrix from the fully connected graph as the impedance between the nodes. The final product is expected to have node attraction/production to be roughly equal and the sum of those should be the number of trips made (Node population*3) It is invoked by the MainFile once so the accuracy for the final model is relatively high as the impact on runtime is low.
Tests: Checked to see if output total trip counts per node are within reasonable limits. In order to ensure this is true, 50 balancing iterations are performed, resulting in trips being balanced and equal to the expected trips (population*3)
Status: Verified & working as intended.

LinkLoad

Description: The LinkLoad module is used to do compute the non-capacitated usage of a link. It is computed by adding trips for all OD pairs that have their shortest path travelling through this specific edge.
Tests: Given a simplified network the expected link loads have been computed and verified against the output of this module.
Status: Verified & working as intended.

genExpansionCandidates

Description: This module generates all candidates for network expansion, Both New unconnected nodes as well as candidates to improve connectivity within the network between nodes are generated here. The output is a structure containing all potential network expansion Candidates

Tests: Given a simplified network the expected candidates for expansion have been determined and verified against the output of this module.

Status: Verified & working as intended.

genCapacityCandidates

Description: This module generates the list of candidates for capacity increase. In order to do so it takes all links where the capacity is not yet maximized. The output is a structure containing all potential Candidates for Capacity Increase

Tests: Expected output of all links with a capacity not yet set to the maximum capacity is easy to verify by comparing the number of links without max capacity against the list as well as ensure that links with a capacity equal to the maximum capacity are not in the list.

Status: Verified & working as intended.

ScoreModule

Description: This module is responsible for scoring the candidates to be evaluated. Effectively the implementation of the CBA technically it contains two functions, calcScore and calcCapScore Both functions calculate the CBA for their respective candidate types. For evaluation purposes / implementation some of the properties are computed after scoring to reduce computational efforts required.

Tests: Score the Candidates for a limited CandidateSet where both types have an expected outcome.

Status: Verified & working as intended.

calcIndicators

Description: Effectively the most important output generation module. This module computes various key performance indicators based on the network state input provided to it. The output are various indicators used to evaluate network growth.

Tests: Simple indicators can be checked for correct values for known inputs. For other inputs only the creation of the parameter has been verified, as well as order of magnitude and sign, not the actual value.

Status: Verified & working as intended.

B.2. Integrated Tested

Module tests are a useful tool to ensure that all individual modules of the model work as intended. However, Modular testing cannot go without an integrated test of multiple modules at the same time and/or all units working together following the conceptual model. In this section the performed integrated tests are explained.

Integration - Single Step

To ensure all modules work and the network will grow, a single iteration step can be performed, if all goes well the network should select an investment candidate based on the inputs given. This effectively sets the time-horizon to 1 step from the network input and the selected investment decision can easily be checked by verifying the outputs of individual modules and then check if the decision matches the expected decision. This is the case even for larger time-horizons.

Expected Behaviour of KPI

Many of the Key Performance Indicators can be roughly predicted in terms of their value over time. For example System Length, average node degree and overall connectivity cannot decrease in a network that is expanding. Simply plotting the KPI will validate these parameters and the behaviour.

Stopping Criterion

Once all the Candidates for expansion and capacity increase stop being beneficial (CBA < 1) Their score is set to -1. If there are no valid investment candidates left, the model should stop and output the KPI and visualisation of the network. Once the model runs without errors this can be checked by selecting a large time horizon and then letting the model stop before the time-horizon ends due to no suitable candidates being found. This check is completed successfully and the model terminates iterations whenever there is no suitable candidate for investment.

Predictable Scenarios

Some Scenarios will have a predictable final network state. For example if there are no costs involved, the full network is expected to be constructed where capacity is equal to or exceeds usage for all links in the network given the demand structure. Similarly a larger speed difference will lead to higher benefits and thus more links being constructed, where a low speed difference or negative speed difference will lead to no investments at all, and the starting network is the final solution.

C

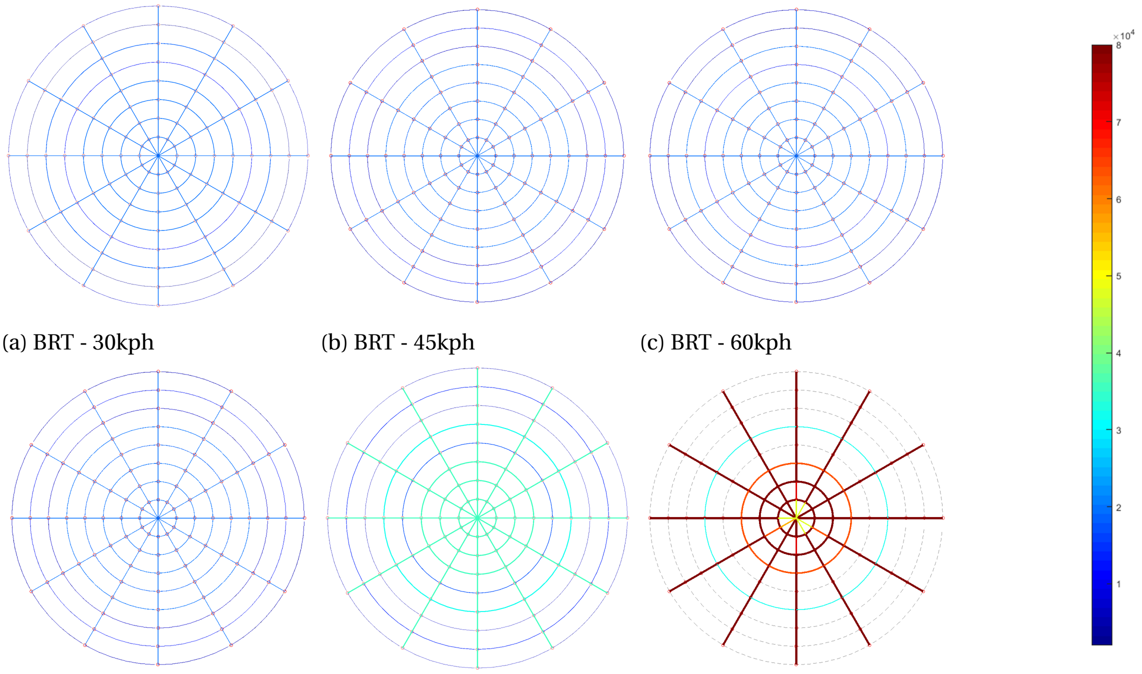
Sensitivity Analysis Results

C.1. Final State Comparisons

KPI	BRT 35	BRT 45	BRT 60	BRT 45	LRT 45	Metro 45	Metro 60
Iterations	1689	1701	1713	1701	1581	1312	1208
β	1,9794	1,9794	1,9794	1,9794	1,9794	1,4845	1,4845
γ	1,00	1,00	1,00	1,00	1,00	0,75	0,7500
Diameter	80	80	80	80	80	80	80
Avg.SP Length	34,50	34,50	34,50	34,50	34,50	35,24	35,14
AvgDeg	3,96	3,96	3,96	3,96	3,96	2,97	2,97
AvgBC	0,0559	0,0559	0,0559	0,0559	0,0559	0,0592	0,0588
σDeg	0,8888	0,8888	0,8888	0,8888	0,8888	1,4892	1,4892
σBC	0,0465	0,0465	0,0465	0,0465	0,0465	0,0477	0,0481
Ringness	0,7020	0,7020	0,7020	0,7020	0,7020	0,4186	0,4597
System Length	1611	1611	1611	1611	1611	826	888
Total Travel Time	2699075	2396325	2114490	2396325	2396325	2470946	2170017
Detour	1,0000	1,0000	1,0000	1,0000	1,0000	1,0275	1,0255
Potential Ridership	4269771	4394750	4502797	4394750	4394750	4363583	4482076
Denied Ridership	4526939	4797376	5026498	4797376	2661376	-1622810	-572934
Avg.Link VoC	2,15	2,21	2,28	2,21	1,34	0,85	0,93
Accum.Score	113600	103579	95899	103579	45125	5199	3975

Table C.1: Sensitivity Analysis - Key Performance Indicator Summary

The table represents the final state key performance indicators for the sensitivity analysis that was performed. The results are grouped by comparison scenario. First 3 rows represent the constant modal costs (BRT) and varying operational speeds (35,45 and 60 kph respectively). the second 3 rows represent the constant operational speed (45kph) and varying modal costs (BRT,LRT,Metro respectively) the last column is the normal Metro outcome for comparison purposes. All of these results are obtained using the Uniform population distribution to make comparison of these indicators "fair". From the resulting table some observations can be made. The BRT mode will always construct a max planar graph. The differences in modal speed does result in a higher potential PT Ridership, and thus in higher denied ridership and link volume over capacity ratios, a clear demonstration of a modal split change due to increased attractiveness of a mode with a higher speed. The results for the 45kph operational speed clearly demonstrate the effects costs have on the network as higher costs lead to a less developed network. The difference between BRT and LRT is marginal in terms of links constructed (both end up with max planar graph) but the larger capacity of LRT and still relatively cheap costs result in a lower denied ridership and average link VoC. Metro scenario has less link connections due to the higher costs, a clear relationship that can be seen from results. Comparing the operational speed of Metro 45 and Metro 60 scenarios also shows that a lower operational speed has a relatively limited effect. Most indicators are similar meaning the resulting networks do not differ that much.

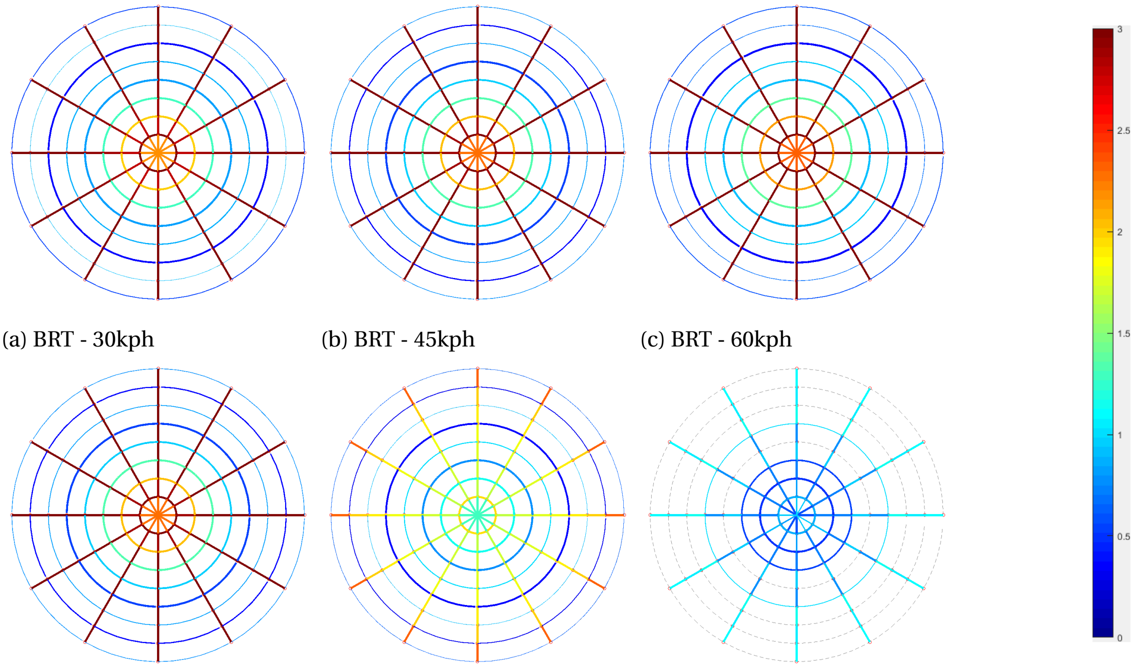


(a) BRT - 30kph (b) BRT - 45kph (c) BRT - 60kph

(d) BRT - 45kph (e) LRT - 45kph (f) Metro - 45kph (g) Scale

Figure C.1: Finalised Network State for each of the scenarios. Colours of the graphs indicate total passengers on a link

The graphs presented in figureC.1 again are grouped by the resulting scenario-groups. First row of 3 is the Bus Rapid Transit comparison for different speeds (30,45,60 respectively). The second group of figures represents constant operational speeds (45kph) and varying modes (BRT,LRT,Metro respectively). Same for the relative link load figure C.2.



(a) BRT - 30kph (b) BRT - 45kph (c) BRT - 60kph

(d) BRT - 45kph (e) LRT - 45kph (f) Metro - 45kph (g) Scale

Figure C.2: Finalised Network State for each of the scenarios. Colours of the graphs indicate Link Loads in final State

C.2. Key Performance Indicator Comparisons

The graphs in this section are similar as in section 5.1. Discussion of these results is done in 5.3.

Beta and Gamma

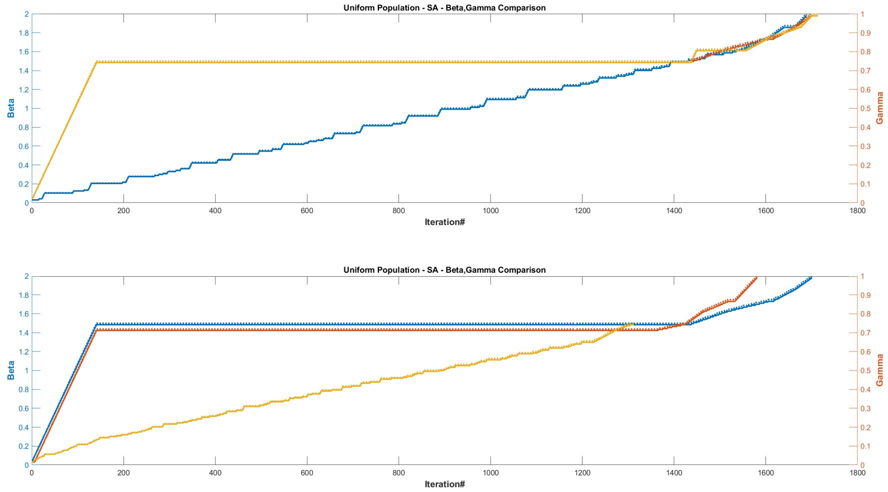


Figure C.3: Comparison of β & γ for various modes given a population distribution

Diameter

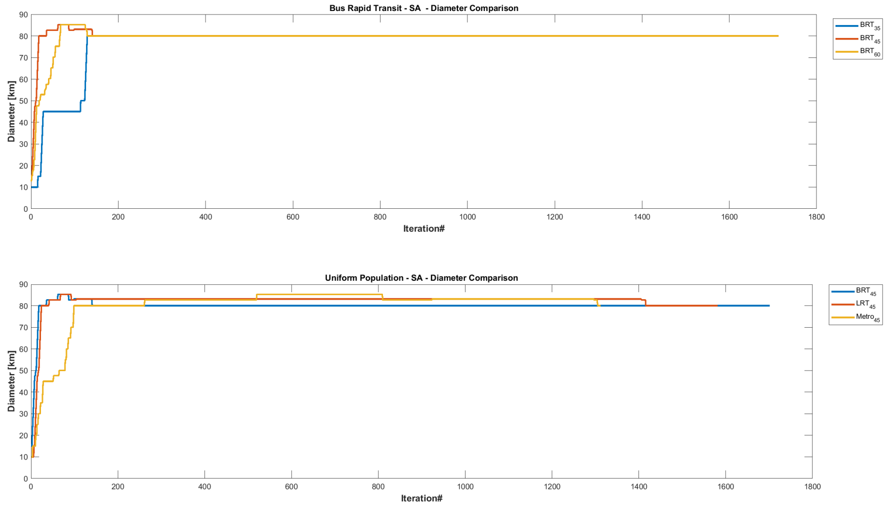


Figure C.4: Comparison of Diameter for various modes given a population distribution

Average Shortest Path Length (APL)

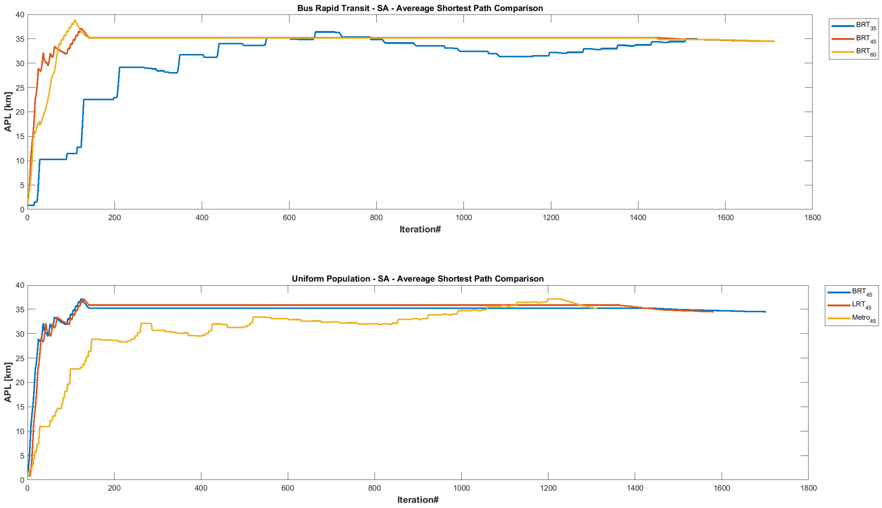


Figure C.5: Comparison of Average Shortest Path Length for various modes given a population distribution

Average Node Betweenness Centrality

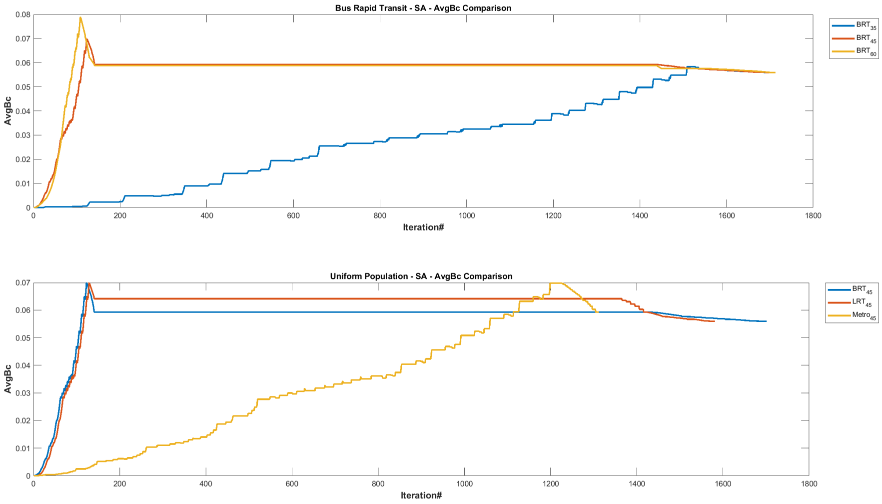


Figure C.6: Comparison of Average Betweenness Centrality for various modes given a population distribution

Ringness

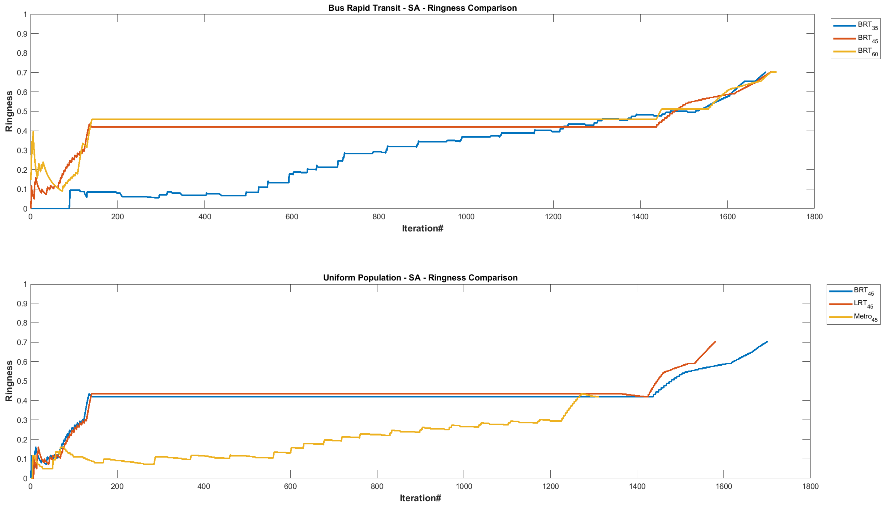


Figure C.7: Comparison of Ringness for various modes given a population distribution

Average Detour Factor

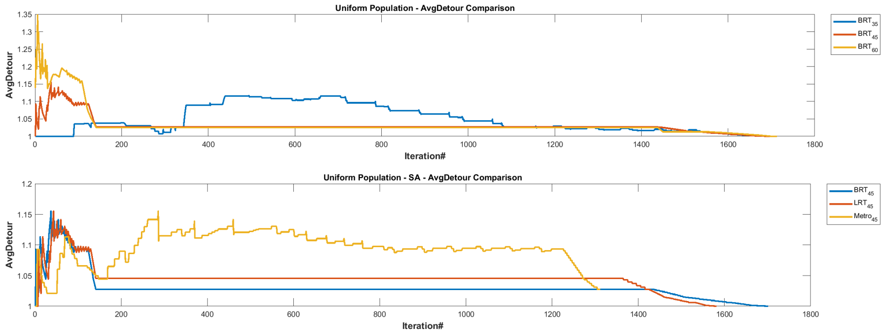


Figure C.8: Comparison of Average Detour Factor for various modes given a population distribution

C.2.1. Performance Indicators

Total System Length

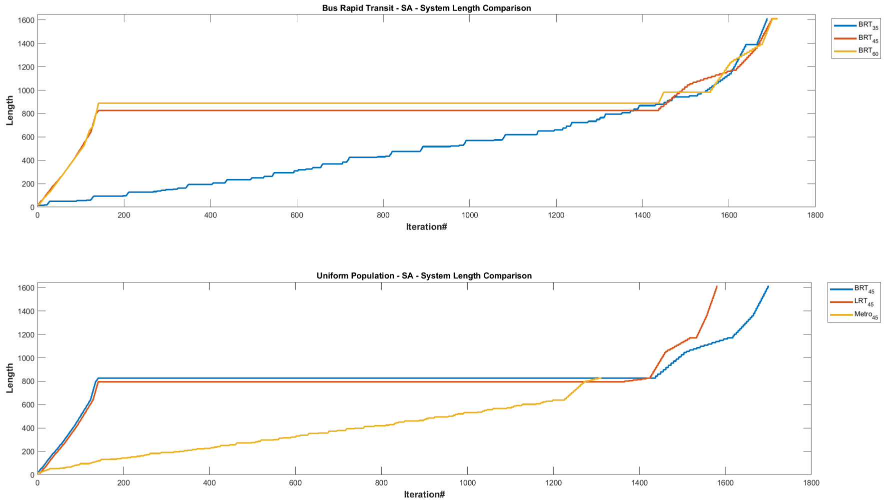


Figure C.9: Comparison of System Length for various modes given a population distribution

Total Travel Time

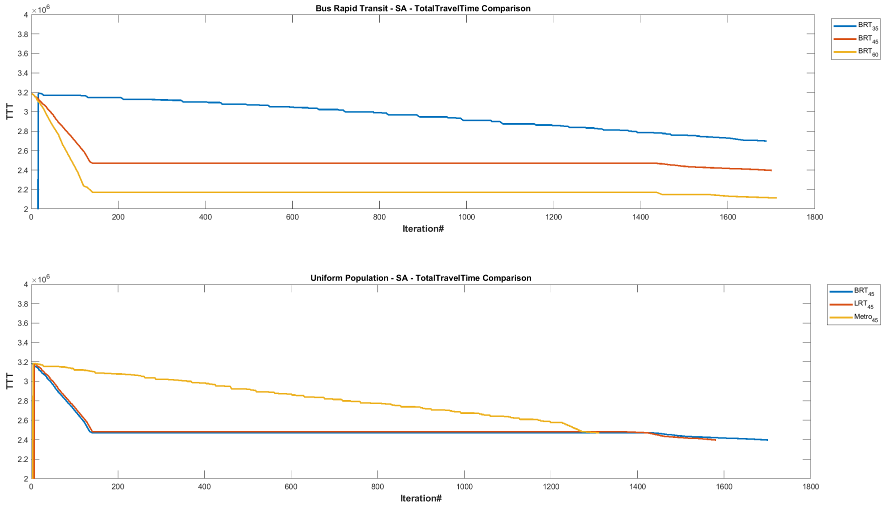


Figure C.10: Comparison of TotalTravelTime for various modes given a population distribution

Potential Ridership

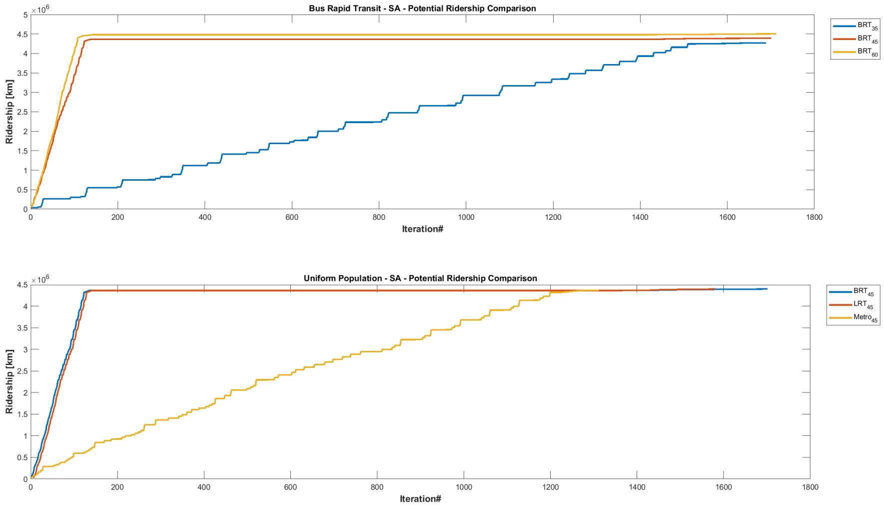


Figure C.11: Comparison of Potential Ridership for various modes given a population distribution

Denied Ridership

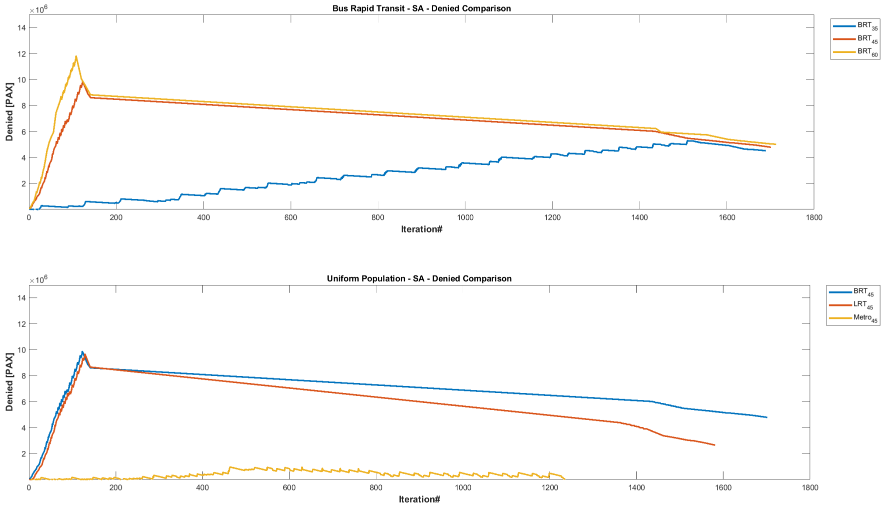


Figure C.12: Comparison of Denied Ridership for various modes given a population distribution

Average Link Volume over Capacity Ratio

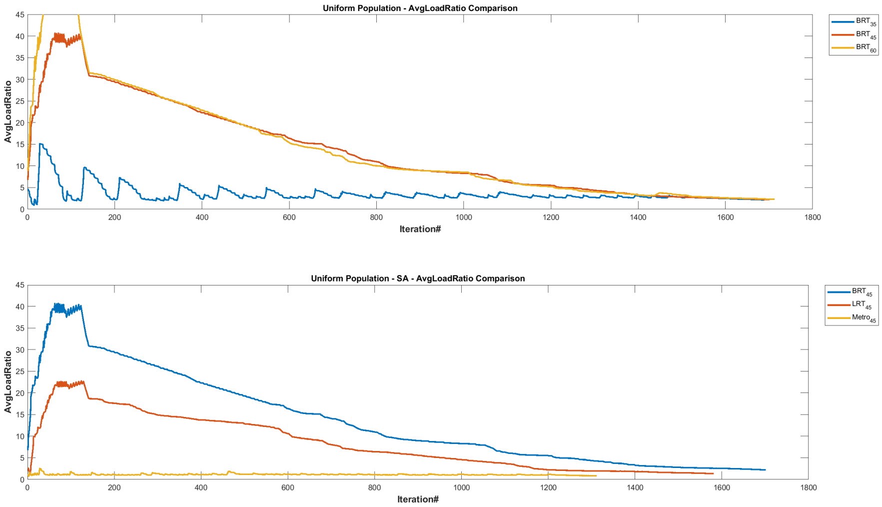


Figure C.13: Comparison of Average Link Volume over Capacity (VoC) for mode parameters given a certain population distribution

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