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Bayesian Inference of Piping Model Uncertainties Based on Field Observations

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Abstract: This paper presents a Bayesian model to determine the model uncertainty of a critical horizontal gradient model for piping for dikes, such a Lane and Bligh. A Bayesian model is needed for two reasons. First, there is a large overlap in cases that failed and survived. Second, the evidence of the failed cases is limited .The model consists of a non-informative prior that is combined with likelihood functions for failed and survived cases. This involves modeling the mean and standard deviation of the model uncertainty as random variables. For survived cases we know the limit state function was larger than 0 for the observed water level. For failed cases we know the limit state function was smaller than 0; or Z = 0; which is a less conservative assumption. This information is used to determine the likelihood functions for failed and survived cases. The prior and likelihoods are combined to find the posterior distributions of the mean and standard distribution of the model uncertainty. Using integration, this finally results in the (lognormal) distribution of the model uncertainty. The model is applied to the data of Bligh and Lane and shows both a high mean and high standard deviation of the model uncertainty, where the model of Lane performs better than Bligh. It is recommended to tailor the proposed model to dikes by making a different distinction between horizontal and vertical erosion. Furthermore, it is recommended to apply the model to more dike specific data since the Bligh data mainly consists of dams instead of dikes.

Keywords: Bayesian inference; backward erosion piping; field observations; model uncertainty.

1 Introduction

1.1 Backward erosion piping

Backward erosion piping, further referred to as piping in this paper, is one of the main failure modes of dikes. Piping occurs in case a head difference over the dike results in uplift of the landside blanket (if applicable) and subsequently in the start of erosion of the granular aquifer at the landside. With sufficient head difference, this erosion progresses below the dike until the river side is reached and failure occurs. There are various models available to determine the required length of the seepages path in order to design sufficiently safe dikes. First, there are the process-based models such as Sellmeijer (Sellmeijer et al. 2011) that are derived from physical processes and calibrated based on lab experiments. These models are typically complex, with many variables that allow for an accurate reconstruction of lab experiments. However, since the required parameters are typically difficult to obtain or exhibit high variability for real dikes, designs based on these models tend to be conservative. The second group of piping models is more empirical as well as more simple. These critical horizontal gradient-based models, see next section, typically relate the required length of the seepage path to a critical head difference by a Creep coefficient. These models have the advantage of simplicity, but are less tailored to local circumstance to allow for design optimization.

1.2 Bligh's piping model

One of the empirical, critical horizontal gradient, piping models is the model of Bligh (Bligh 1915), which uses a so-called creep coefficient to determine the critical horizontal gradient over a dike. Lane's model is an elaboration of Bligh's model (Lane 1935), with the difference that Lane's model differentiates between horizontal and vertical parts of the seepage path.

$$L > \Delta H \cdot C \tag{1}$$

The creep coefficient (C) shows the critical horizontal gradient for a structure. In both Bligh's and Lane's model, C is intended as a safe value to relate the occurring vertical head difference (ΔH) over a dike or dam to the required length of the seepage (L) . The creep factor is based on a dataset of failed and survived dams, mainly masonry dams on sand foundations. The creep factor depends on the type of sand, where fine sand is deemed less resistant to piping than coarse sand. However, the model uncertainty of this model is unknown and therefore it is not possible to quantify the safety of structures that are designed using these rules or to design and assess new structures with explicit safety criteria.

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The goal of this paper is to present a method to quantify the model uncertainty in Bligh's creep ratio and to apply this model to Bligh's original data. For more information and details, please refer to Kanning (2012).

1.3 Bligh's piping data

The data used by Bligh for his piping model, are shown in Figure 1. In this figure, both the failed and survived cases of Bligh (1915), as published in the paper of Lane (1935) are plotted for sand, fine sand and silt, together with the safe Creep ratio of 18. Bligh originally used a ratio of 15, later this is adapted to 18 to obtain more conservative design (TAW 2002). It should be noted that the majority of cases are (masonry) dams and not dikes. It can be noted in the graph that there are both unexpected failures (i.e., a failure below the Creep line) as well as unexpected survivals (i.e., survivals far above the Creep line). The line was intended as a conservative value for design purposes. In order to overcome this overlap, a Bayesian model is proposed in this paper. Furthermore there is typically limited data available of the cases (the only thing we know is that the limit state function was equal to or below 0 for failures); this inequality data is best treated in a Bayesian way.

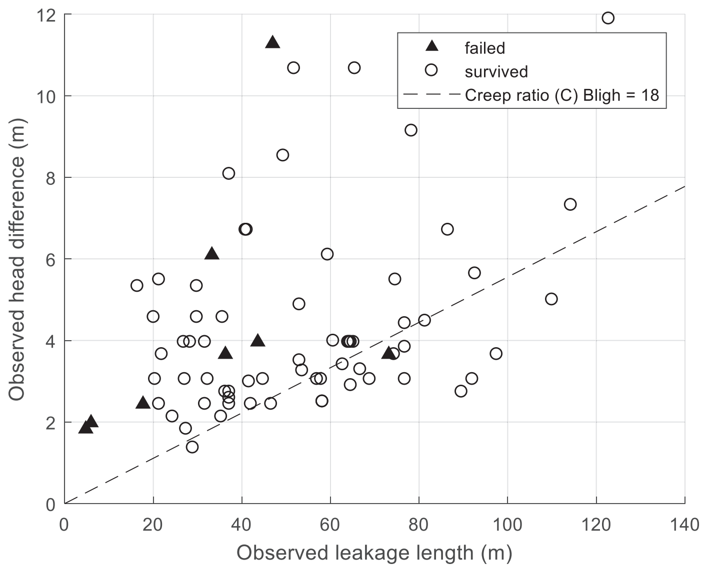

Figure 1. The data of Bligh for sand, fine sand and silt, failed and survived, plotted against the proposed Creep ratio C= 18.

2 Bayesian Inference Method

The used Bayesian inference method to determine the model uncertainty (θ) in Bligh's model is briefly discussed in this section. More details of the method are provided in Kanning (2012).

2.1 Method

The first step in the method is to define a limit state function (Z) including model uncertainty (θ), see Eq. (2). This equation is based on Eq. (1). In Eq. (2), ΔH_c is the critical water level as calculated using Eq. (1) and ΔH is the occurring water level. The resistance term ΔH_c is modelled as *L/C*, in which *1/C* is replaced by *c'*.

$$Z = \theta \cdot \Delta H_c - \Delta H = \theta \cdot \frac{L}{C} - \Delta H = \theta \cdot c' \cdot L - \Delta H \tag{2}$$

Since both the mean (μ_θ) and standard deviation (σ_θ) of θ are unknown; these are modeled as random variables. A normal distribution for both variables appeared to give the most stable results, see Eq. (3).

$$\begin{aligned} \mu_\theta &= N(\mu_{\mu_\theta}, \sigma_{\mu_\theta}) \\ \sigma_\theta &= N(\mu_{\sigma_\theta}, \sigma_{\sigma_\theta}) \end{aligned} \tag{3}$$

The posterior distribution of μ_θ and σ_θ given the evidence (*D*) is derived from by Bayes' rule as Eq. (4).

$$p(\mu_\theta, \sigma_\theta \mid D) = N \cdot P(D \mid \mu_\theta, \sigma_\theta) \cdot p(\mu_\theta, \sigma_\theta) \tag{4}$$

In this equation, $p(\mu_\theta,\sigma_\theta \mid D)$ is the posterior distribution, $P(D \mid \mu_\theta,\sigma_\theta)$ is the likelihood function, $p(\mu_\theta,\sigma_\theta)$ is the prior distribution and N is a normalizing constant. The evidence D is the dataset containing failed and survived piping data. The prior is discussed in more detail in Section 2.2, the likelihood function in Section 2.3. The likelihood function is determined by combining the likelihood function for the survived cases with the likelihood function for the failure cases. The distribution of the model uncertainty θ can subsequently be determined by Eq. (5):

$$p(\theta \mid D)=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} p(\theta \mid \mu_\theta,\sigma_\theta)\,p(\mu_\theta,\sigma_\theta \mid D)\,d\mu_\theta\, d\sigma_\theta \tag{5}$$

In this Eq. (5), $p(\theta \mid \mu_\theta,\sigma_\theta)$ is the probability density of θ given μ_θ and σ_θ. In this paper, the prior and all the likelihood functions of Eq. (4) are numerically combined. Numerical integration is used to solve Eq. (5). In Section 3, examples of the outcomes are presented for the data of Bligh (1915).

2.2 Prior distribution

A non-informative prior for a problem with unknown mean and standard deviation as we are dealing with in this paper is given by e.g. Box and Tiao (1992) as:

$$p_{non\text{-}inf}(\mu_\theta,\sigma_\theta) \propto \frac{1}{\sigma_\theta} \qquad for -\infty < \mu_\theta < \infty \tag{6}$$

In the case of Bligh's data, a uniform distribution might also be justified since we know most data is obtained in relatively extreme circumstances. This would limit the domain, and hence justify a uniform prior. Since the non-informative prior of Eq. (6) results in the most conservative results, this prior is chosen throughout this paper.

2.3 Likelihood functions for survived cases

The first step in determining the likelihood is to write a survival observation i as a ratio of load divided by resistance (ξ_{surv}), which is an approximation of the observed model performance, see Eq. (7).

$$\xi_{surv,i}=\frac{\Delta H_i}{c' L_i} \tag{7}$$

Hence, the observation D consists of ξ_{surv} and ξ_{fail} for failed cases. In case of a survival observation, the limit state function value is observed to be larger than 0. Typically, the leakage length L is known, as well as the head difference ΔH that is at least survived. Applying this to Eq. (2), combined with Eq. (7)

$$\theta-\xi_{surv,i}>0 \tag{8}$$

For a normal distribution of θ the likelihood function in case of a survival observation can subsequently be written as:

$$L_{surv}(\mu_\theta,\sigma_\theta)=\prod_{i=1}^{n}\left(1-\Phi\left(-\frac{\xi_{surv,i}-\mu_\theta}{\sigma_\theta}\right)\right) \tag{9}$$

2.4 Likelihood functions for failed cases

In case of an observed failure, it is more challenging how to interpret the observed data. The leakage length is typically relatively well known by design drawings or adjacent sections. However, the observed water level is more challenging to interpret. In case of dam failures, the observed water level at failure is likely the exact water level where the limit state function is 0, as after failure the water level will drop again. For dikes however, it is possible for the water level to keep rising after a breach. Either way, it is known that the limit state function was exceeded, resulting in Eq. (10).

$$\theta-\xi_{fail,i}<0 \tag{10}$$

In case the exact water level at failure is known, the "<" sign is replaced by a "=", which results in a different likelihood function that is less conservative. This is further elaborated in Kanning (2012), but not further considered in this paper. There is also a possibility that the observed water level is lower than the water level at failure, especially in case of limit available data and no available hydrographs. This option is not further considered in this paper. Similar to the previous section, Eq. (10) can be used to derive the likelihood function in case of failures according to Eq. (11)

$$L_{fail}(\mu_\theta,\sigma_\theta)=\prod_{j=1}^{m}\left(\Phi\left(-\frac{\xi_{fail,j}-\mu_\theta}{\sigma_\theta}\right)\right) \tag{11}$$

The combined likelihood $p(\theta \mid \mu_\theta,\sigma_\theta)$ is finally found by combining Eq. (9) and Eq. (11):

$$P(D \mid \mu_\theta,\sigma_\theta)=L_{surv}(\mu_\theta,\sigma_\theta)\cdot L_{fail}(\mu_\theta,\sigma_\theta)=\prod_{i=1}^{n}\left(1-\Phi\left(-\frac{\xi_{surv,i}-\mu_\theta}{\sigma_\theta}\right)\right)\cdot\prod_{j=1}^{m}\left(\Phi\left(-\frac{\xi_{fail,j}-\mu_\theta}{\sigma_\theta}\right)\right) \tag{12}$$

3 Application to Bligh's Data

The method of Section 2 is subsequently applied to Bligh's data for sand, fine sand and silt, since this is supposedly the most sensitive to piping for dikes. The data consists of 8 failures and 65 survivals, see Figure 1. The joint distribution of μ_θ and σ_θ is shown in Figure 2, which is based on Eq. (4). It can be seen that the mean and standard deviation are relatively large and exhibit large uncertainty. This is mostly explained by the wide scatter in the data and both unexpected failures and unexpected survival cases. The distribution of θ given the observations can finally be obtained by applying Eq. (5) using numerical integration. The distribution of θ shows a high mean (6.4) and variance (3.8). This would imply that a design made using this equation has a probability of failure of 0.08.

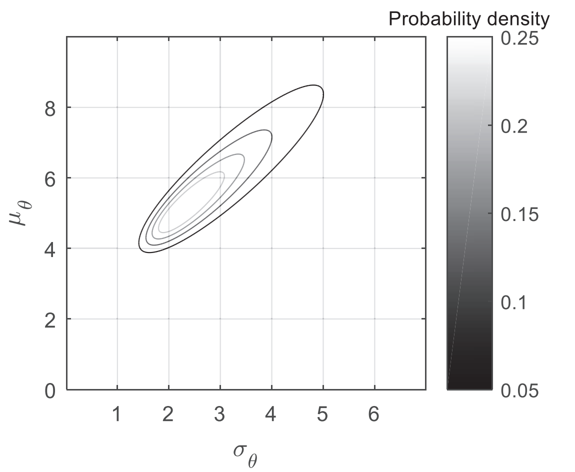

Figure 2. Joint probability distribution of μ_θ and σ_θ based on Bligh's data for sand, fine sand and silt.

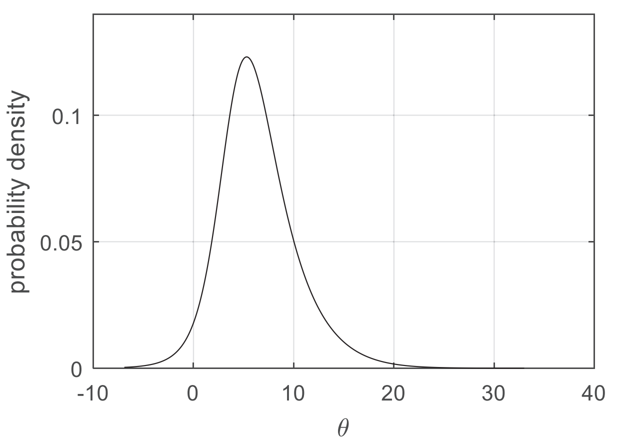

Figure 3. Distribution of model uncertainty θ based on Bligh's data for sand, fine sand and silt.

4 Results for Lane's Model

Lane (1935) observed that the vertical part of the seepage path was more important than the horizontal part. Based on this, he proposed an weighted creep ratio, where the horizontal length only accounts for 1/3 compared to the vertical length. Together with this, he proposed a modified safe Creep ratio of 8.5. Applying this model to the data results in Figure 4.

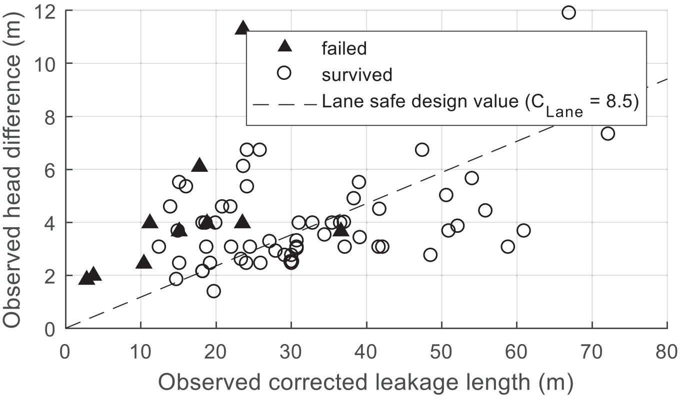

Figure 4. The data of Bligh, using Lane's model for sand, fine sand and silt, plotted against the proposed Creep ratio by Lane (C_{Lane} 8.5)

Compared to Bligh's model, this results already in less unexpected survivals and a better relation between model and observations. When applying Lane's model to the Bayesian method of this paper, a mean of the model uncertainty of 3.5 if found and a standard deviation of 1.9. This is already considerably better than Bligh's model.

5 Discussion and Conclusions

This paper presents a Bayesian method that allows for the explicit quantification of piping model uncertainty for the rules of Bligh and Lane, based on failed and survived cases. The method allows for the incorporation of failures and survivals, which is needed especially for piping for which failure observations are limited and modeling is subject to high uncertainties. The approach is applied to the piping model and data of Bligh (1915). This data shows a high mean and standard deviation of the model uncertainty. The model of Lane is an improvement of Bligh's model, which is expressed in less model uncertainty.

The data used in this paper mostly stem from (masonry) dams, not dikes. The data typically have a significant vertical part of the seepage path, which is not entirely representative for dikes that are typically dominated by the horizontal length. However, this vertical erosion through the blanket layer is important (see e.g. USACE 1956) and requires different modeling than the horizontal piping erosion. Hence, for future research it is recommended to separate the horizontal and vertical erosion for critical horizontal gradient-based models for dikes. Furthermore, since most cases in this paper are from dams, it is recommended to apply the method to dike specific observations, which should result in model uncertainty.

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