The Influence of the Increasing Penetration of Photovoltaic Generation on Integrated Transmission-Distribution Power Systems

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Chapter 2
The Influence of the Increasing Penetration of Photovoltaic Generation on Integrated Transmission-Distribution Power Systems

Maria Kootte and Cornelis Vuik

Abstract Power system simulations should be adapted to be applicable to the trends that are currently evoked by the energy transition. This transition is pushing our power system from a traditional hierarchical system to a modern interactive system. In order to keep the supply and transport of energy safe and reliant, we need to change the way we perform power system simulations. This requires a comprehensive framework in which both transmission and distribution systems are simultaneously analyzed. This chapter describes how transmission and distribution networks are modeled together as an integrated network and used to do steady-state operation analysis in order to assess the interaction of these two networks. Furthermore, we investigate the influence of the increasing amount of imbalance at distribution level on the transmission network that is evoked by the increase of highly variable resources and loads at distribution level. This influence is not taken into account in traditional power system simulations as power networks are analyzed on its own. We show that the hybrid network representation is a powerful tool to analyze modern power systems and that the effects of increased PV penetration under normal operating conditions are limited.

1 Introduction

Electrical power systems are responsible for generation, transmission, and distribution of electrical power. In traditional power flow computations, the transmission and distribution networks are modeled and solved separately, where each system operator is responsible for safe operation and planning of their own electrical power system. Both transmission and distribution system operators (TSO and DSOs) use steady-state power flow computations to obtain insight into the state of their net-

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work [1]. The electrical power system is a large interconnected system, containing one transmission network and multiple distribution networks. In order to separately analyze these networks, TSOs model the distribution network as an equivalent load bus, while DSOs model the transmission network as an equivalent generator bus [2]. These assumptions were justified as bulk power was generated at transmission level and transported hierarchically toward end consumers such as households and industries.

However, the electricity landscape is changing. A major driver for this change is the rapid increase of renewable resources such as wind and photovoltaic (PV) power, connected at distribution level. These resources are also called distributed energy resources (DERs) [2]. The changes induce bi-directional power flow between transmission and distribution network and increased imbalance of the electrical power system [3]. Transmission networks are, in general, balanced networks while distribution networks are, in general, unbalanced networks. This imbalance is a result of different phase loading levels and untransposed distribution cables [4]. The main driver for the increase of DERs is the high level of PV power penetration in urban residential areas. More and more households are motivated to place PV panels on their rooftop out of care for the planet, financial incentives, and/or lowering purchase and installation costs [5]. The government supports this trend as it is in line with the global sustainable development goals, but it puts extra pressure on the electricity grid as it will be difficult for grid operators to maintain safe voltage levels.

PV panels can be classified as residential and utility panels. Utility panels can be used to replace traditional bulk power plants at transmission level. Residential panels are small and located at distribution level as they are mainly placed on rooftops in urban areas. Residential PV panels produce active power only as voltage regulatory purposes not allow them to produce reactive power [5]. This can lead to a voltage drop in the network to an undesirable level. On top of that, as distribution networks are unbalanced networks, a voltage drop can result in different phase variations. This can again result in a certain degree of imbalance that is harmful for the network [6]. DSOs operate their system between a certain bandwidth of allowed voltage levels and degree of imbalance [7].

Increased PV penetration at distribution level can also affect transmission systems. Traditional steady-state analysis, done on separate electrical power systems, is not able to capture the effects that the networks have on each other [3]. Therefore, there is a need for integrated power flow simulations that is capable of analyzing the effects that these networks have on each other. In this work, we are going to present a hybrid integrated transmission-distribution network model as a single entity. This hybrid network model represents the transmission network as a single-phase network model, while the distribution network is represented by a three-phase network model [8]. The interface description is not the only challenge that should be considered when doing integrated power flow computations. Power flow solvers have been developed so far for separate power flow analysis [4]. These methods cannot one-on-one be translated to run power flow computations on integrated network models. It is important to consider the efficiency and robustness of the existing separate methods when they are used to solve the integrated power flow problem.

The integrated model of an electrical power system can be used to study the sensitivities toward PV power penetration such as steady-state voltage stability, induced imbalance, and reduced power losses. The results are compared with a full three-phase representation of the integrated network. This chapter contains the following contributions: (1) a description of steady-state power flow computations in (balanced) transmission and (unbalanced) distribution networks (Sect. 2); (2) a modeling approach on how these separate networks can be integrated as a hybrid network model and which techniques can be used to solve this system (Sect. 3); (3) a description of photovoltaic power generation and how it is connected to an electrical power system (Sect. 4); and (4) results of the simulation using a varying amount of PV penetration compared to a zero PV load base case and a full three-phase representation of the integrated network (Sect. 5), after which we draw conclusions (Sect. 6).

2 Steady-State Power Flow Computations

The steady-state power flow problem is the problem of determining the voltages V in a network, given the specified power[1] $S = P + \iota Q$ and current I [9]. V and I are related by Ohm's Law, $I = YV$, and S and V are related by $S = VI^*$. Y represents the admittance of a power cable. Power is generated in three phases leading to three sinusoidal functions that describe phase a, b, and c of the voltage and of the current. Current is never specified in an electricity system, and therefore we substitute Ohm's Law into $S = VI^*$ and get a nonlinear equation for S, which is the three-phase nonlinear power flow equation, described as follows:

$$S_p = V_p(\mathbf{Y}V)_p^*, \quad p \in \{a, b, c\}. \tag{1}$$

We represent an electricity network as a graph consisting of buses $i = 1, \ldots, N$ representing generators, loads, and shunts and branches representing transformers and cables. During steady-state power flow computations we determine the voltages V_i of each bus given the power supply and demand S_i of each bus and admittance Y_{ij} of each branch.

2.1 Transmission Networks

The transmission network is the high-voltage network, responsible for the transportation of power over large distances. It is a balanced system which means that the magnitude of the voltage ($|V|$) of the three phases a, b, and c are equal and that the phase shift (ϕ) between the voltages are equal [9]. This means that $|V|_a = |V|_b = |V|_c$ and $\phi_{ab} = \phi_{bc} = \phi_{ca} = \frac{2}{3}\pi$. To simplify and speed up the computations in the transmis-

[1] We use the subscript ι as the imaginary unit.

sion network, only V_a is calculated. The other two phases are deducted from phase a.

The problem of finding the voltages V_i given power S_i and Y_{ij} simplifies then to the following single-phase relationship, where a represents the phase:

$$S_i^a = V_i^a \overline{I}_i^a = V_i^a (\overline{Y\mathbf{V}})_i^a = V_i^a \sum_{k=1}^{N} \overline{Y}_{ik}^a \overline{V}_k^a. \tag{2}$$

We use Newton-Raphson power mismatch (NR power) to compute unknown quantities at each bus i [9]. NR power computes V_i using the following power mismatch formulation:

$$\Delta S_i = S_{s,i} - S(V_i) \approx 0. \tag{3}$$

S_s is the specified power, the known information at generator and load nodes: $S_s = S_g - S_d$, subscript g representing the generator buses and d the load buses. $S(V)$ is the injected power, $S(V) = V(\mathbf{Y}V)^*$. The complex power S is split into an active and reactive part and combined to form the power mismatch vector F,

$$\mathbf{F}(\mathbf{x}) = \begin{bmatrix} \Delta \mathbf{P} \\ \Delta \mathbf{Q} \end{bmatrix} = \begin{bmatrix} \mathbf{P}_s - \mathbf{P}(\mathbf{x}) \\ \mathbf{Q}_s - \mathbf{Q}(\mathbf{x}) \end{bmatrix} = \mathbf{0}, \tag{4}$$

where $\mathbf{x}$ represents the state variables $x_i = \begin{bmatrix} \delta_i & |V_i| \end{bmatrix}^T$ which form the voltage in the phasor notation $V_i = |V| \exp{(\iota\delta)}_i$.

We compute V in an iterative manner using the Jacobian J of the power mismatch vector:

$$\Delta \mathbf{x}^\nu = -J^{-1}(\mathbf{x}^\nu)\mathbf{F}(\mathbf{x}^\nu), \tag{5}$$

$$\mathbf{x}^{\nu+1} = \mathbf{x}^\nu + \Delta \mathbf{x}^\nu, \tag{6}$$

where the Jacobian is represented as follows: $J(\mathbf{x}) = \begin{bmatrix} \frac{\partial P}{\partial \delta} & \frac{\partial P}{\partial |V|} \\ \frac{\partial Q}{\partial \delta} & \frac{\partial Q}{\partial |V|} \end{bmatrix}$.

We repeat this until the norm of the power mismatch vector $|F|_\infty$ is lower than a certain tolerance value ε. We choose $\varepsilon = 10^{-5}$ and start with a flat profile as initial guess: $V^0 = 0$. The Newton-Raphson algorithm is described in algorithm 1.

2.2 Distribution Networks: Three-Phase Representation

Distribution systems are unbalanced: the three phases are not equal in magnitude nor in phase shift [10]. This requires us to compute all the three phases in every iteration when solving equation (1). We use Newton-Raphson Three-Phase Current Injection

Algorithm 1 The Newton-Raphson iterative method

1: Set $\nu = 0$ and choose appropriate starting value $\mathbf{x}^0$;
2: Compute $F(x^\nu)$;
3: Test convergence: If $|F(x^\nu)|_\infty \leq \varepsilon$ then x^ν is the solution, otherwise continue;
4: Compute the Jacobian matrix $\mathbf{J}(x^\nu)$;
5: Update the solution:

$$\Delta x^\nu = -\mathbf{J}^{-1}(x^\nu)F(x^\nu)$$
$$x^{\nu+1} = x^\nu + \Delta x^\nu;$$

6: Update iteration counter $\nu + 1 \rightarrow \nu$, go to step 2.

Method (NR-TCIM) [11] to solve distribution networks. Instead of applying the standard Newton-Raphson method to power mismatches, Ohm's law is directly used resulting in the current mismatch vector:

$$\mathbf{F}(\mathbf{x}) = \begin{bmatrix} \Delta \mathbf{I}^{Re,abc}(\mathbf{x}) \\ \Delta \mathbf{I}^{Im,abc}(\mathbf{x}) \end{bmatrix} = \begin{bmatrix} \mathbf{I}_s^{Re,abc} - \mathbf{I}^{Re,abc}(\mathbf{x}) \\ \mathbf{I}_s^{Im,abc} - \mathbf{I}^{Im,abc}(\mathbf{x}) \end{bmatrix}. \tag{7}$$

The specified current I_s and computed current $I\,(V)$ are calculated using the injected complex power and Ohm's law:

$$I_{s,i} = \left(\frac{\overline{S_s}}{V}\right)_i \quad \text{and} \quad I(V)_i = \mathbf{Y}V_i. \tag{8}$$

The Jacobian is formed by taking the derivative of the real and imaginary current mismatch with respect to the real and imaginary voltage.

2.3 Network Components

As explained earlier in this section, an electrical power system is represented as a graph containing branches and buses. These branches and buses represent the several elements that are connected in a network. Loads, shunts, and generators are expressed as buses, while general cables and transformers are modeled using equivalent branches. The buses in an electrical power network are either a load bus (PQ bus), a generator bus (PV bus), or a slack bus, depending on the information we know at that point. The loads in a network are modeled as PQ buses, loads consume power and at this bus, the active (P) and reactive power (Q) are specified. Generators are modeled as PV buses, except for the first generator in a network, this bus is modeled as the slack bus. Generators supply power and at this bus, the active power (P) and voltage magnitude ($|V|$) are specified. At a slack bus, the voltage magnitude and angle are specified. The several bus types and the according information are summarized in Table 1.

Table 1 Bus types in a network and the information we know and not know at each bus i

Bus type	Known	Unknown
PQ bus	$P_i,\ Q_i$	$\delta_i,\ \lvert V_i \rvert$
PV bus	$P_i,\ \lvert V_i \rvert$	$Q_i,\ \delta_i$
Slack bus	$\delta_i,\ \lvert V_i \rvert$	$P_i,\ Q_i$

In separate network analysis, the distribution network is modeled as an equivalent load bus in transmission power computations, while the transmission network is modeled as the slack bus in distribution power computations.

The exact configuration for loads, shunt, transformers, and regulators installed along distribution cables can be found in the more extensive version of the authors [12].

3 Integrated Transmission and Distribution Network Model

In order to analyze the power system, we need an integrated transmission and distribution network simulation tool. It is not straightforward to integrate these separate domains, because of the unbalanced nature of distribution networks compared to the balanced transmission network. Next to the modeling issues, one should also regard the difficulties that can arise when solving the integrated system. The differences between transmission and distribution power systems have led to the development of different solvers. Especially the high R/X ratio on distribution lines and the radial configuration compared to the meshed configuration of transmission networks have influenced the development of the solvers [4]. As explained in previous section, we use NR power to solve transmission networks and NR-TCIM to solve distribution networks. Integrating these two separate systems into a new domain should consider the integrated design and carefully investigate which solver is most advantageous to solve the integrated power flow problem.

In order to integrate the two separate networks we need an interface that is capable of integrating a hybrid network (a single-phase network with a three-phase network) as a single entity [8]. The two networks are physically connected by a substation. The substation transforms high-voltage power of transmission networks to low-voltage power by a couple of in-series transformers. In separate distribution network analysis, this substation is modeled as a single transformer that connects the distribution network from the right-hand side, and where the transmission network is represented as an equivalent generator. This substation transformer is modeled as a branch using an equivalent Π-model.

In order to create an integrated network, we need an interface capable of modeling the substation transformer in a hybrid network configuration such that it connects a

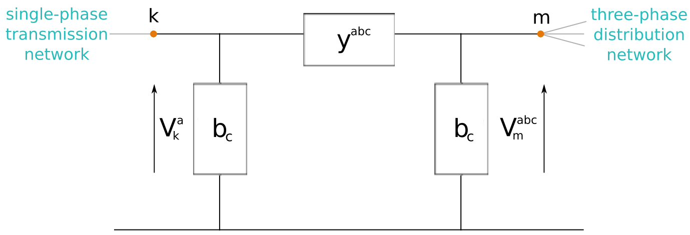

Fig. 1 The substation transformer in a hybrid network connecting single-phase bus k and three-phase bus m

single-phase and a three-phase network model. The substation transformer modeled as an equivalent Π-branch connecting the hybrid network is depicted in Fig. 1. It connects bus k of the transmission network with bus m of the distribution network.

The MonoTri formulation [13] is an interface that describes how the substation transformer should be modeled in order to integrate the hybrid network as a single entity. It couples the single-phase quantities at the transmission side to the three-phase quantities at distribution side by transforming the nodal admittance matrix $\mathbf{Y}_{km}$ of the substation transformer. The coming subsection explains how this is established.

3.1 The MonoTri Formulation as Interface to Integrate the Hybrid Network Model

To establish the connection of bus k and m via the admittance matrix $\mathbf{Y}_{km}$, we use three transformer matrices:

$$\mathbf{T_1}, \quad \mathbf{T_3} = \frac{1}{3}[1\ a\ a^2], \quad \mathbf{T_4} = \frac{1}{3}\,[1\ 1\ 1]\,, \quad \text{and} \quad \mathbf{T_5} = \frac{1}{3}\left[1\ a^2\ a\right], \quad a = e^{\frac{2}{3}\pi\iota}.$$

This transformation is based on the assumption that the connecting bus k is perfectly balanced. This means that the single-phase and three-phase quantities are related as follows:

$$\left[V_a\ V_b\ V_c\right]_k^T = \mathbf{T_1}\left[V_a\right]_k, \tag{9}$$

$$\left[I_a\right]_k = \mathbf{T_3}\left[I_a\ I_b\ I_c\right]_k^T, \tag{10}$$

$$\left[S_a\right]_k = \mathbf{T_4}\left[S_a\ S_b\ S_c\right]_k^T. \tag{11}$$

The exact outlook of the interface depends on whether the unified system is solved using NR-Power or NR-TCIM. Relations (9), (10), and (11) are substituted in the corresponding power flow equations. These outlooks are described in paragraphs given in Sects. 3.1.1 and 3.1.2.

3.1.1 MonoTri Formulation Using Current Injections

The NR-TCIM method uses Ohm's law directly. The relation between node k and m is expressed as follows:

$$I = \mathbf{Y}V \quad \Leftrightarrow \quad \begin{bmatrix} I_k \\ I_m \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} V_k \\ V_m \end{bmatrix}. \tag{12}$$

If node k and m were both modeled in three phase, we know the following:

$$I_k^{abc} = \mathbf{Y}_{11}^{abc} V_k^{abc} + \mathbf{Y}_{12}^{abc} V_m^{abc}, \tag{13}$$
$$I_m^{abc} = \mathbf{Y}_{21}^{abc} V_k^{abc} + \mathbf{Y}_{22}^{abc} V_m^{abc}. \tag{14}$$

We now multiply Eq. (13) by $\mathbf{T_3}$ to obtain I_k^a:

$$I_k^a = \mathbf{T_3}\mathbf{I}_k^{abc} = \mathbf{T_3}\mathbf{Y}_{11}^{abc} V_k^{abc} + \mathbf{T_3}\mathbf{Y}_{12}^{abc}\mathbf{V}_m^{abc}. \tag{15}$$

We then substitute $\mathbf{V}_k^{abc}$ in Eqs. (15) and (14) by $\mathbf{T_1} V_k^a$ (Eq. 9):

$$I_k^a = \mathbf{T_3}\mathbf{I}_k^{abc} = \mathbf{T_3}\mathbf{Y}_{11}^{abc}\mathbf{T_1} V_k^a + \mathbf{T_3}\mathbf{Y}_{12}^{abc}\mathbf{V}_m^{abc}, \tag{16}$$
$$\mathbf{I}_m^{abc} = \mathbf{Y}_{21}^{abc}\mathbf{T_1} V_k^a + \mathbf{Y}_{22}^{abc} V_m^{abc}. \tag{17}$$

From (16) and (17), we see that our new nodal admittance matrix becomes

$$\mathbf{Y}_{km} = \begin{matrix} & 1 & 3 \\ 1 & \mathbf{T_3}[\mathbf{Y}_{11}^{abc}]\mathbf{T_1} & \mathbf{T_3}[\mathbf{Y}_{12}^{abc}] \\ 3 & [\mathbf{Y}_{21}^{abc}]\mathbf{T_1} & \mathbf{Y}_{22}^{abc} \end{matrix}_{km}. \tag{18}$$

3.1.2 The MonoTri Formulation Using Power Injections

We can also start from the power equations. The relation between node k and m is expressed as follows:

$$S = VI^* \quad \Leftrightarrow \quad \begin{bmatrix} S_k \\ S_m \end{bmatrix} = \begin{bmatrix} V_k \\ V_m \end{bmatrix} \begin{bmatrix} I_k \\ I_m \end{bmatrix}^*. \tag{19}$$

In the same manner as current injections, we can write this relation in three phase:

$$
\begin{aligned}
S_k^{abc} &= V_k^{abc} I_k^{abc*} + V_k^{abc} I_m^{abc*}, && (20)\\
S_m^{abc} &= V_m^{abc} I_k^{abc*} + V_m^{abc} I_m^{abc*}. && (21)\\
&\Leftrightarrow && (22)\\
S_k^{abc} &= \mathrm{diag}(V_k^{abc}) \cdot (\mathbf{Y}_{kk}^{abc} V_k^{abc})^* + \mathrm{diag}(V_k^{abc}) \cdot (\mathbf{Y}_{km}^{abc} V_k^{abc})^*, && (23)\\
S_m^{abc} &= \mathrm{diag}(V_m^{abc}) \cdot (\mathbf{Y}_{mk}^{abc} V_k^{abc})^* + \mathrm{diag}(V_m^{abc}) \cdot (\mathbf{Y}_{mm}^{abc} V_m^{abc})^*. && (24)
\end{aligned}
$$

We multiply the first line from the left by $\mathbf{T_4}$ to obtain S_k^a:

$$
S_k^a = \mathbf{T_4} S_k^{abc} = \mathbf{T_4}\mathrm{diag}(V_k^{abc}) \cdot (\mathbf{Y}_{kk}^{abc} V_k^{abc})^* + \mathbf{T_4}\mathrm{diag}(V_k^{abc}) \cdot (\mathbf{Y}_{km}^{abc} V_m^{abc})^*. \quad (25)
$$

Then, we substitute $V_k^{abc} = \mathbf{T_1} V_k^a$ (Equation (9)) in Equations (24) and (25) and obtain the following:

$$
\begin{aligned}
S_k^a &= \mathbf{T_4}\mathrm{diag}(\mathbf{T_1} V_k^a) \cdot (\mathbf{Y}_{kk}^{abc} \mathbf{T_1} V_k^a)^* + \mathbf{T_4}\mathrm{diag}(\mathbf{T_1} V_k^a) \cdot (\mathbf{Y}_{km}^{abc} V_m^{abc})^*, && (26)\\
S_m^{abc} &= \mathrm{diag}(V_m^{abc}) \cdot (\mathbf{Y}_{mk}^{abc} \mathbf{T_1} V_k^a)^* + \mathrm{diag}(V_m^{abc}) \cdot (\mathbf{Y}_{mm}^{abc} V_m^{abc})^*. && (27)
\end{aligned}
$$

We can rewrite $\mathbf{T_4}\mathrm{diag}(\mathbf{T_1} V_k^a)$, the first part of the RHS in (26), as

$$
\begin{aligned}
\mathbf{T_4}\mathrm{diag}(\mathbf{T_1} V_k^a) &\Leftrightarrow \mathbf{T_4}\mathrm{diag}(\mathbf{T_1})\mathrm{diag}(V_k^a) && (28)\\
&= \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & a^2 & 0 \\ 0 & 0 & a \end{bmatrix} \mathrm{diag}(V_k^a) && (29)\\
&\Leftrightarrow \underbrace{\frac{1}{3}\begin{bmatrix} 1 & a^2 & a \end{bmatrix}}_{\mathbf{T_5}} \mathrm{diag}(V_k^a) && (30)\\
&\Leftrightarrow \mathrm{diag}(V_k^a)\mathbf{T_5}. && (31)
\end{aligned}
$$

This results in the following relations for single-phase and three-phase power:

$$
\begin{aligned}
S_k^a &= \mathrm{diag}(V_k^a) \cdot (\mathbf{T_5}\mathbf{Y}_{kk}^{abc} \mathbf{T_1} V_k^a)^* + \mathrm{diag}(V_k^a) \cdot (\mathbf{T_5}\mathbf{Y}_{km}^{abc} V_m^{abc})^*, && (32)\\
S_m^{abc} &= \mathrm{diag}(V_m^{abc}) \cdot (\mathbf{Y}_{mk}^{abc} \mathbf{T_1} V_k^a)^* + \mathrm{diag}(V_m^{abc}) \cdot (\mathbf{Y}_{mm}^{abc} V_m^{abc})^*. && (33)
\end{aligned}
$$

Equations (32), (33) yield the following transformed admittance matrix $\mathbf{Y}_{km}$:

$$
\mathbf{Y}_{km} = \begin{array}{c} \\ 1 \\ 3 \end{array} \begin{array}{c} \begin{array}{cc} 1 & 3 \end{array} \\ \begin{bmatrix} \mathbf{T_5}[\mathbf{Y}_{kk}^{abc}]\mathbf{T_1} & \mathbf{T_5}[\mathbf{Y}_{km}^{abc}] \\ [\mathbf{Y}_{mk}^{abc}]\mathbf{T_1} & \mathbf{Y}_{mm}^{abc} \end{bmatrix}_{km} \end{array}. \quad (34)
$$

3.1.3 The Full Three-Phase Representation

In order to compare the results of the hybrid network configuration using the MonoTri formulation of the interface, we need an integrated network that is modeled in a full three-phase representation. The full three-phase formulation is able to capture the possibly induced imbalance at transmission level due to the high increase of PV power penetration at distribution level. The hybrid representation of the integrated transmission-distribution network is not capable of showing this imbalance and thus might exhibit errors in the simulation results. This is because the hybrid network only shows one phase and therefore the imbalance between phases is not visible.

The full three-phase representation does not require an interface description but a three-phase representation of the transmission network. This description is based on the assumption that the transmission system is in a balanced initial state, and only possible imbalance that can arise is due to the increased level of PV penetration. In a balanced network, the phases b and c are deducted from the first phase a. We transform the voltage V_a, the complex power S_a, and the admittance Y_a of all the transmission buses and branches to their three-phase equivalents. We use the following transformer matrices for every bus i in the network:

$$\mathbf{T_1} = \begin{bmatrix} 1 & a^2 & a \end{bmatrix}^T \quad \text{and} \quad \mathbf{T_2} = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix}^T, \quad a = e^{\frac{2}{3}\pi\iota},$$

and identity matrix $\mathbf{I}_{3\times3}$. This results in the following:

$$\mathbf{T_1}\,[V_a]_i = \begin{bmatrix} V_a & V_b & V_c \end{bmatrix}_i^T, \tag{35}$$

$$\mathbf{T_2}\,\begin{bmatrix} S_a \end{bmatrix}_i = \begin{bmatrix} S_a & S_b & S_c \end{bmatrix}_i^T, \tag{36}$$

$$\begin{bmatrix} Y_{11}^a \otimes \mathbf{I}_{3\times3} & Y_{12}^a \otimes \mathbf{I}_{3\times3} \\ Y_{21}^a \otimes \mathbf{I}_{3\times3} & Y_{22}^a \otimes \mathbf{I}_{3\times3} \end{bmatrix}_{ij} = \begin{matrix} 3 \\ 3 \end{matrix}\overset{\begin{matrix} 3 & \quad 3 \end{matrix}}{\begin{bmatrix} \mathbf{Y}_{11}^{abc} & \mathbf{Y}_{12}^{abc} \\ \mathbf{Y}_{21}^{abc} & \mathbf{Y}_{22}^{abc} \end{bmatrix}_{ij}}. \tag{37}$$

3.2 *Solvers for Integrated Hybrid Systems*

The MonoTri formulation of the interface connects the two separate network models into one integrated hybrid network model. As a next step, we consider which techniques can be used to solve this integrated hybrid system efficiently. Newton-Raphson solvers have different outlooks when they are used to solve transmission systems and distribution systems. Newton-Raphson using power mismatches is better capable of solving transmission systems, while the Newton-Raphson method using three-phase current injections (NR-TCIM) is better capable of solving distribution systems, as explained in Sect. 2. Nevertheless, the authors of [14] compare several formulations of Newton-Raphson solvers for both transmission and distribution systems. Although they show that the NR-TCIM formulation works best for distribution systems, it is also finely capable of solving transmission systems. As general electrical power systems consists of multiple distribution feeders compared to one transmission feeder and knowing the fact that distribution networks are often much larger than transmission networks, the distribution feeder will dominate the design of the integrated system. Using the outcome of [14], we expect that NR-TCIM should be capable of solving the integrated system. Nevertheless, we would like to point out that it is interesting to analyze which solvers are most efficient and robust to solve integrated systems.

4 The Impact of Photovoltaic Penetration on Electrical Power Systems

Now that we have defined how we can integrate an electrical power system, we are interested in the effects of increased PV penetration on integrated transmission and distribution networks. For safe operation and planning of an electrical power system, it is important to maintain steady-state voltage stability among all buses in the networks and to limit the amount of imbalance of the network [15]. This means that individual voltage levels may not exceed limits prescribed by system operators. For transmission systems, this is usually between 0.95 and 1.05 pu. The maximum amount of imbalance is usually defined as 5% [7].

A hybrid integrated network model is not capable of showing the amount of imbalance on the transmission part of the network as it only represents one phase of the transmission network. Therefore, we validate our computations using a full three-phase presentation of the integrated network. In this manner, we can see how close the approximations of the hybrid network formulation are. If we see that the amount of imbalance is relatively high on the transmission network, the proposed MonoTri formulation is not suitable to analyze integrated systems. The authors of [16] propose a technique to incorporate the imbalance of the transmission network using the MonoTri interface without running full three-phase computations. The incorporation of this work is subject to future research.

Another implication of increased amount of PV penetration is bi-directional power flow on a system that is originally designed for one-way power flow [17]. Necessary regulation measures should be taken to limit this. A positive consequence that can arise due to increased use of DERs is that these are often installed along distribution feeders close to the loads. This can reduce the power losses as the distance between generator and consumer significantly decreases [18]. These are all important effects that could be analyzed using an integrated network model. In this chapter, we focus our study on investigation of steady-state voltage stability and amount of imbalance on distribution and transmission networks.

4.1 Photovoltaic Panels

Two types of PV panels can be connected to an electrical power system: (1) residential rooftop PV systems and (2) utility-scale PV panels. As the name implies, residential rooftop systems can be found on rooftops of households and buildings. This means that these systems are connected at distribution level, closely located to the loads. Utility-scale panels are installed on transmission level and distribution level and—due to the size of these installations—could be used to replace conventional power generators [18].

The focus of this study is on residential rooftop systems, closely connected to loads along the distribution feeder. The location close to the loads might limit the power losses that arise due to the distances that electrical power conventionally have to bridge to get from a transmission power plant toward end consumers at distribution level. On the contrary, as residential rooftops only supply active power, the location close to the loads might lead to insufficient supply of reactive power to the loads [18]. This can cause the voltage to drop. This will not only lead to voltage variations outside the safe operating limits, but also to individual phase variations as the distribution network is unbalanced. This leads to increasing imbalance, which is harmful for the entire electrical power systems [6]. Therefore, care should be taken to investigate the possible effects on transmission level by analyzing integrated power systems with an increasing amount of PV penetration.

4.1.1 PV Power Models

Most of the residential PV systems provide mainly active power and are therefore modeled as a negative load (PQ) bus, containing only negative active power injection. We are going to analyze the steady-state behavior of the integrated network by running simulations with various levels of PV penetration. Several methods exist to define the amount of PV penetration. We use the definition based on the total available generation in the base case [18], defined as follows:

$$\text{PV penetration}(\%) = \frac{\text{total PV generation}}{\text{total generation}} \cdot 100\% \tag{38}$$

We use varying penetration levels between 10% and 50%. Based on the data analysis of [17], we see that the peak irradiation[2] of residential PV systems is 7 kW/m^2 and that an individual panel with this level of irradiation produced a maximum power of 105 W. In this way, we can calculate how many panels are necessary to obtain the desired amount of PV penetration and whether this number of panels is a realistic amount for a certain neighborhood or residential area.

[2] Irradiance is an instantaneous measurement of solar power over some area [19].

4.2 PV Penetration Scenarios

We are going to run power flow simulation on integrated systems using two artificially created integrated networks from available IEEE data that can be used for test simulations. The focus of this study is to simulate the effects of increasing PV penetration on steady-state voltage stability and amount of imbalance on both distribution and transmission networks. We are defining five scenarios with varying levels of PV penetration which we are going to assess:

0. The base case: No PV penetration.
I. 10% PV penetration.
II. 20% PV penetration.
III. 50% PV penetration.
IV. An extreme scenario of 200% PV penetration, only supplied via phase A.

We are going to do steady-state power flow simulations using the Matpower[3] library. We use a single core machine with an Intel Core i7-7600 processor, 2.80 GHz CPU, and 8.00 GB memory.

4.3 Test Cases

We create integrated test cases from the existing transmission and distribution test cases from the Matpower library and resources page of IEEE Power & Energy Society [21]. We use the 9-bus and 118-bus networks from Matpower as balanced transmission network test cases. We use the IEEE 13-bus and 123-bus data from IEEE P&ES as unbalanced distribution test cases. This results in two integrated test cases: the T9-D13 network and the T118-D123 network. We model the substation transformers in these networks in a Delta-Grounded Wye configuration. We also create an integrated test case of a transmission network with multiple distribution networks in order to increase the effects that distribution networks can have on transmission networks. A country has most of the time one transmission network with several distribution networks connected to it. This is thus a realistic test case. This results in a third test case: T118-10D123, the 118-bus transmission network connected to ten 123-bus distribution networks.

In total we have three test cases of a transmission network integrated with one or multiple distribution networks:

1. T9-D13.
2. T118-D123.
3. T118-10D123.

[3] MATPOWER is a package of free, open-source MATLAB-language M-files for solving steady-state power system simulation and optimization problems [20].

The PV panels are connected to these networks at one bus that is modeled as a negative PQ-bus. Figure 2 shows such an integrated network. This is the visualization of the T9-D13 network, where the PV panels are connected to bus 21 of the integrated network.

5 Simulation Results

In this section, we are going to analyze the steady-state voltage stability and amount of transmission and distribution imbalance on the three test cases using the five different scenarios. We start with the steady-state voltage stability.

5.1 Steady-State Voltage Stability

The steady-state voltages of distribution networks should not exceed certain predefined limits. For the 13-bus distribution network, this limit is defined as 10%. The amount of PV penetration, modeled as a negative PQ-load only supplying active power, can reduce the voltage as the amount of reactive power is reduced. The following figures show the new steady-state voltages supplying the distribution network with different amount of PV penetration.

Figures 3, 4, and 5 show that during normal PV penetration levels, the voltages of the T9-D13 test case never exceed its limits. Furthermore, it shows that the voltage profile is staying close to its original profile of the base case (without any PV penetration). Only in the extreme scenario, we see that the levels drop significantly.

The results of the bigger test cases, T118-D123 and T118-10D123, are summarized in Table 2. This table shows the minimum and maximum voltage magnitudes of the three separate phases. This table shows that also in the bigger test cases, the voltage magnitudes do not drop on to a certain extend that it exceeds the prescribed operating limits. The test case including multiple distribution networks, the T118-D123 test case, is exceeding its maximum safe operating limit. But, this is already happening during the zero PV penetration level. This has more to do with the amount of distribution networks than to the effect of PV penetration. The authors of [17] studied the effects of several levels of PV penetration on distribution networks only. They also concluded that the effects on voltage levels are limited. The results of this study are thus in line with their findings.

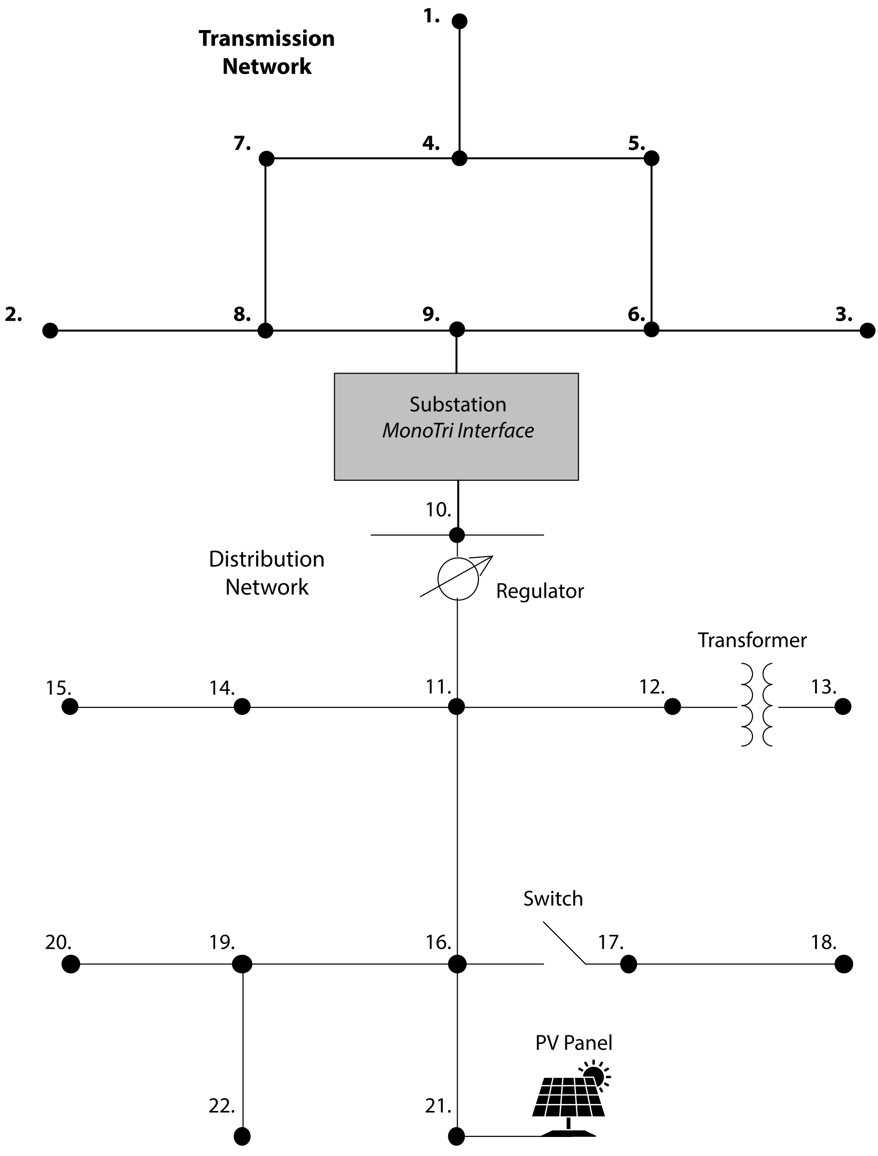

Fig. 2 The T9-D13 test case visualized. The upper part is the 9-bus meshed transmission network. It is connected at bus 9 via the MonoTri interface description to bus 10, which is the original slack bus of the distribution network. The special elements are visualized at their location. The residential rooftop PV panel is located at bus 21 of the integrated network

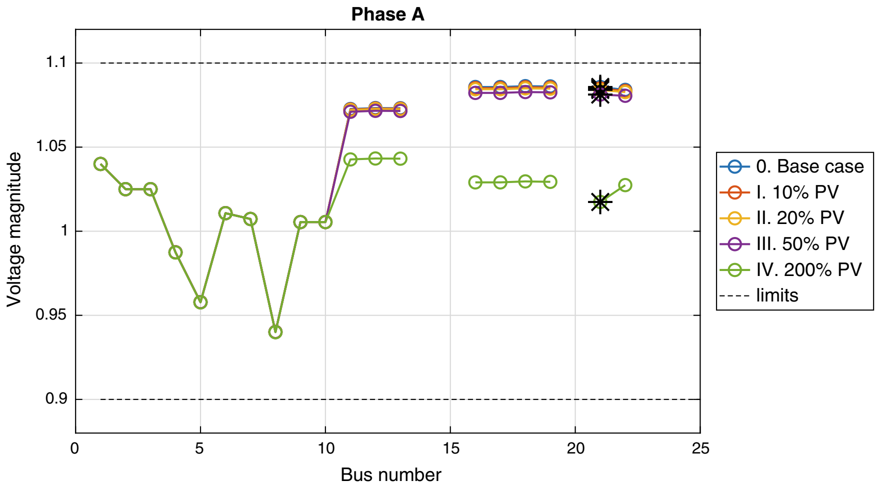

Fig. 3 Steady-state voltage profile of phase A of the T9-D13 network having different levels of PV penetration. The operating limits, which the voltages should not exceed, are given. The black star shows the location of the bus to which the PV panels are connected. The missing information on certain buses is due to the fact that the distribution feeder contains single-, double-, and three-phase laterals

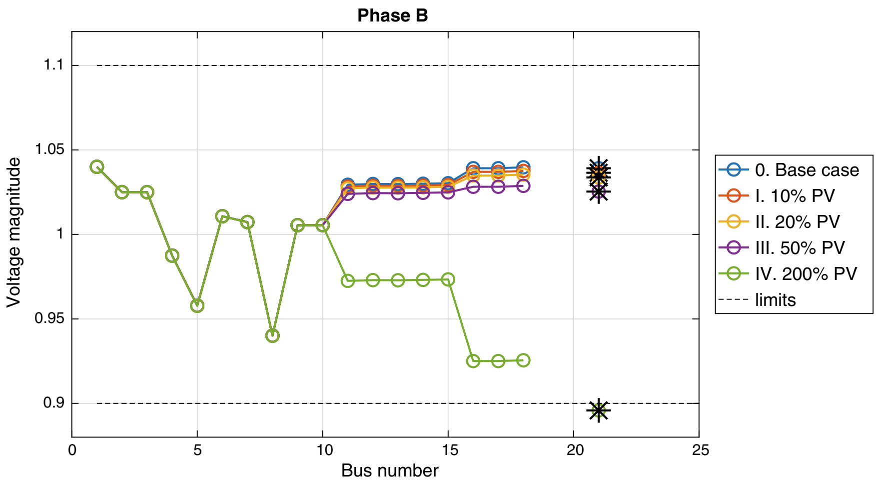

Fig. 4 Steady-state voltage profile of phase B of the T9-D13 network having different levels of PV penetration. The operating limits, which the voltages should not exceed, are given. The black star shows the location of the bus to which the PV panels are connected. The missing information on certain buses is due to the fact that the distribution feeder contains single-, double-, and three-phase laterals

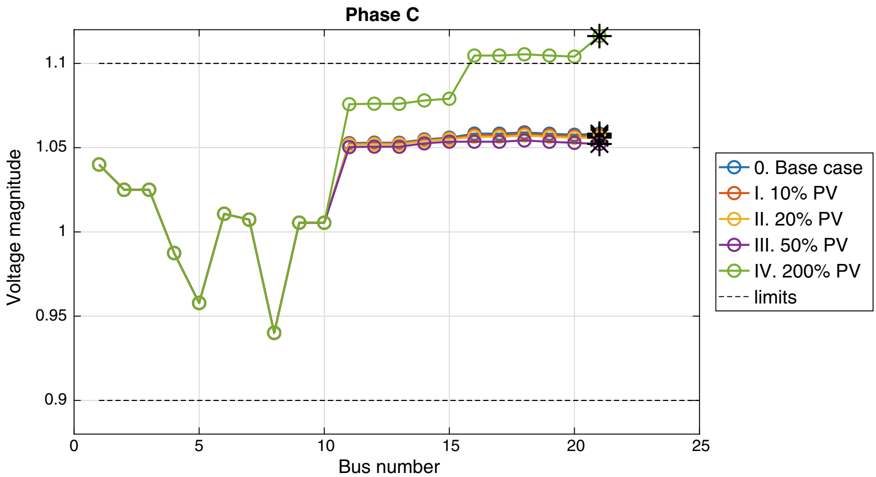

Fig. 5 Steady-state voltage profile of phase C of the T9-D13 network having different levels of PV penetration. The operating limits, which the voltages should not exceed, are given. The black star shows the location of the bus to which the PV panels are connected. The missing information on certain buses is due to the fact that the distribution feeder contains single-, double-, and three-phase laterals

Table 2 The minimum and maximum voltage magnitudes of phases a, b, and c. Test cases T118-D123 and T118-10D123 are shown. The boldface printed magnitudes exceeds the voltage magnitude limits of distribution test cases

Scenario	T118-D123						T118-10D123					
	V^a_{min}	V^a_{max}	V^b_{min}	V^b_{max}	V^c_{min}	V^c_{max}	V^a_{min}	V^a_{max}	V^b_{min}	V^b_{max}	V^c_{min}	V^c_{max}
0.	0.97	1.08	0.96	1.03	0.97	1.06	0.96	**1.12**	0.96	1.07	0.96	**1.10**
I.	0.97	1.08	0.97	1.03	0.97	1.05	0.96	**1.12**	0.96	1.07	0.96	**1.10**
II.	0.97	1.08	0.97	1.03	0.97	1.05	0.96	**1.12**	0.96	1.07	0.96	1.09
III.	0.97	1.07	0.97	1.03	0.97	1.05	0.96	**1.12**	0.96	1.07	0.96	1.09
IV.	0.97	1.04	0.91	1.01	0.97	1.13	0.96	1.08	**0.90**	1.05	0.96	**1.17**

5.2 Amount of Imbalance on Distribution and Distribution Networks

The amount of PV penetration can lead to increased imbalance on both distribution and transmission networks. Distribution networks are already unbalanced network, due to the unequal mutual coupling between phases on the lines, different voltage drop of the three phases, and unbalanced loads installed along the distribution feeder. PV power penetration can increase the amount of imbalance. The National Electrical Manufactures Association (NEMA) uses the following definition for the amount of imbalance [7]:

Table 3 The amount of maximum and average voltage unbalance in percentage of the distribution feeder for the three different test cases and the number of iterations of the three different methods

Test case	Distribution					Transmission		
	0.	I.	II.	III.	IV.	0.	III.	IV.
T9-D13	2.33	2.40	2.47	2.69	11.29	0.04	0.04	0.36
T118-D123	2.15	2.19	2.19	2.27	10.99	0.03	0.04	0.27
T118-10D123	2.17	2.21	2.21	2.27	11.14	0.10	0.11	0.67

$$V_{unb} = \frac{MaxDevfromV_{average}}{V_{average}} * 100\% \tag{39}$$

The recommended standard under normal steady-state conditions is that the voltage unbalance of distribution systems will not exceed 5% [7], while the amount of imbalance on transmission systems should be as minimal as possible. Table 3 shows the amount of imbalance on distribution networks for the defined scenarios on three test cases. We also show the amount of imbalance on transmission networks, which we can calculate from running full three-phase computations. We only show this amount for the base case, scenario. IV: 50% PV penetration, and the extreme scenario.

Table 3 shows that the amount of imbalance on distribution networks is staying within a limit of 5% when penetrating the networks with a standard amount of PV penetration. Also the amount of imbalance on transmission networks is limited. Only when penetrating the networks with an extreme amount of PV penetration, we see that the imbalance increase beyond its limits. Even then, the amount of imbalance on transmission networks is limited. This means that as long the amount of PV penetration is staying within its operating limits, hybrid network formulation is sufficient for doing integrated power flow computations. As soon as the amount of PV penetration reach its critical limits, measures need to be taken at distribution level as the imbalance increases already beyond its limits and probably won't influence transmission networks.

The authors that introduce the MonoTri formulation [16] also assess the amount of unbalance on transmission networks. They show as well that the amount of unbalance is limited under certain levels of PV penetration. The conclusions are thus in line with this work.

6 Conclusion

The rapid increase of photovoltaic (PV) power generation at distribution level makes it important to analyze electrical power systems as integrated systems. Integrated systems are necessary to analyze the steady-state voltage levels of transmission and distribution networks simultaneously including the interaction that these networks have on each other. As transmission networks are in general balanced networks and therefore modeled in single phase, while distribution networks are unbalanced and modeled in three phase, we need an interface that is capable of modeling this hybrid network configuration as an integrated network. The MonoTri formulation is a description of the interface that is able to integrate these two separate networks.

The increasing amount of residential PV panels connected near the loads at the distribution network can lead to several effects which can be harmful for both the distribution and transmission networks. Residential rooftop panels are, in general, not capable of supplying reactive power. The lack of reactive power can lead to a voltage drop of the steady-state voltages and exceeding the limits that are designed on safe operating conditions. Next to that, increased PV penetration can increase the amount of imbalance due to the design of the distribution network. The amount of imbalance can also have harmful consequences for electrical power systems and should therefore not exceed a certain limit. Transmission networks are in general balanced and thus modeled as a single-phase network. They are therefore not able to show the amount of imbalance. The increased PV penetration could eventually lead to induced imbalance on transmission networks, which cannot be shown using a hybrid (single-phase/three-phase) network design.

We have run several steady-state power flow simulations on integrated transmission-distribution networks to investigate whether the voltages exceed its safe operating limits and induce imbalance on distribution and transmission networks. We used these simulations to analyze the effects of increasing amount of PV penetration by comparing various levels of PV penetration with a base case of zero PV power. We used standard penetration levels of 10, 20, and 50 %, and an extreme case of 200 %. In the extreme case, PV power is only injected through phase A. This is done to intensify the effects that PV penetration can have on transmission networks.

The simulations show that under normal levels of PV penetration, the steady-state voltages of distribution networks slightly drop but never exceed the safe operating limits. The amount of imbalance increases slightly, but not until an extent that would be harmful for distribution networks. Only in the extreme scenario, we see effects on the transmission network. We see a slight drop of voltage magnitude in some buses and a tiny amount of extra imbalance compared to the base case. Yet, this extreme scenario is already so harmful for distribution networks that measures should be taken at this level to prevent harmful amount of imbalance or voltage drop. We expect that these preventive measures will also decrease the effects on the transmission network. This means that the hybrid network representation of the integrated system is sufficient to analyze the steady-state behavior of electrical power systems.

To summarize, we can conclude that a hybrid network model is a sufficient tool to analyze the effects of increased PV penetration of integrated electrical power systems. The amount of induced imbalance and the increased voltage drop are still within safety margins according to distribution operating standards and transmission networks are not affected during these normal scenarios. The extreme case shows a slight increase in imbalance, but as the effects on distribution networks are much worse, we expect that preventive measures will be taken at distribution level such that this will not influence transmission networks.

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