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# An automated system to determine constitutive model parameters from in situ tests

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ABSTRACT: Selecting an appropriate soil constitutive model and determining the corresponding model parameters for numerical analysis are considered most challenging in geotechnical engineering. While many empirical relationships have been proposed to derive soil parameters from in situ test results, there is no clear procedure on how to derive model parameters uniquely. In practice, available data during the early stages of projects is often limited to field test data. Consequently, different engineers provide different numerical solutions for the same problem. As a solution, the authors present a proof of concept for an automated parameter determination (APD) system, using concepts of graph theory to determine constitutive model parameters from in situ tests while keeping the system transparent (verifiable) and adaptable (extendable). The study aims to increase the confidence in parameter determination for numerical analysis by giving the user of the system, the geotechnical engineer, control over the system. Using a spreadsheet of parameters and equations as input, the system generates paths between the parameters and calculates the parameter values for coarse-grained soil, starting from CPT data. Further validation and tweaking of the system, as well as the extension to other types of soils, are part of ongoing research.

RÉSUMÉ: La sélection d'un modèle constitutif approprié d'un sol et la détermination de ses paramètres pour une analyse numérique sont considérées comme des tâches difficiles en géotechnique. Bien que de nombreuses relations empiriques aient été proposées pour identifier les paramètres du sol à partir des résultats d'essais in situ, il n'y a pas de procédure claire sur la façon de déterminer les paramètres d'une loi de comportement de manière unique. En pratique, les données disponibles au cours des premières étapes d'un projet sont souvent limitées aux données d'essais sur le terrain. Par conséquent, différents ingénieurs fournissent différentes solutions numériques pour un même problème. En guise de solution, les auteurs présentent une démonstration de faisabilité pour un système de détermination automatique des paramètres (APD), utilisant des concepts de la théorie des graphes pour déterminer les paramètres de modèles constitutifs à partir d'essais in situ, tout en gardant le système transparent (vérifiable) et adaptable (extensible). L'étude vise à accroître la confiance dans la détermination des paramètres pour l'analyse numérique en donnant à l'utilisateur du système, l'ingénieur géotechnicien, le contrôle du système. En utilisant en entrée des paramètres et des équations dans une feuille de calcul, le système génère des chemins entre les paramètres, et calcule les valeurs de ceux-ci pour dessols à gros grains à partir des données d'un CPT. La validation et l'ajustement ultérieur du système, ainsi que l'extension à d'autres types de sols, font partie des recherches en cours.

KEYWORDS: Automated parameter determination (APD); empirical correlations; constitutive modelling; in situ testing; soil characterisation; cone penetration test (CPT)

#### 1 INTRODUCTION

The past three decades computer technologies have improved and have resulted in a wider use of numerical methods in geotechnical practice. At the same time, constitutive models have developed significantly making it possible to simulate complex soil behaviour (e.g., stress-, strain- and time-dependency). Despite the presence of these advanced models, engineers still tend to favour less sophisticated models, such as the linear elastic perfectly-plastic Mohr-Coulomb model. The Mohr-Coulomb model is used for its simplicity and suitability for many applications. Its number of input parameters is limited to five, whereas more complicated models require a larger number of input parameters (e.g., 13 parameters for the Hardening Soil Small-Strain model) that have to be defined from a larger number of experimental tests (in situ tests and laboratory tests) which is not always possible due to lack of data.

In situ tests, such as the cone penetration test (CPT), are popular among engineers since they are quick and reproducible for site characterisation, soil profiling and estimating constitutive properties of the soil with minimal disturbance and at low cost, unlike laboratory tests like oedometer tests and triaxial tests. The main issue with interpreting in situ test results is the large amount of empiricism engineers have to rely on; parameters cannot be determined directly from experimental curves as in laboratory testing, but through empirical relationships, and often many relationships are available to determine the same parameter, resulting in a wide range of answers for the same problem. The outcome of each parameter depends on the validity and limitations of the selected method, and interpretation is based on engineering judgement. Over the years, many authors have developed empirical relationships to interpret soil from in situ test results, such as Kulhawy & Mayne (1990), Lunne, Powell & Robertson (1997) and Robertson (2015). The challenges in soil interpretation from experimental data have increasingly led to the demand for a more efficient parameter determination system to perform more reliable numerical simulations. *"There is little point in doing a refined analysis if the properties cannot be identified clearly"* (Graham, 2006).

To address the problem described above, the past two decades artificial neural networks (ANNs) have been increasingly used in geotechnical practice (Reale, Gavin, Libric, & Juric-Kacunic, 2018). By capturing the relationship between input and output parameters in a network without any prior knowledge, ANNs can mimic human brain behaviour and work with incomplete information. The main issue with ANNs is their lack of transparency; they are known as "black boxes" that are incapable of explaining how information is used to derive a solution.

This study presents a proof of concept for an automated parameter determination (APD) system from in situ test results, using concepts from graph theory. Key aspects are *transparency*, i.e., information used by the system should be verifiable, and *adaptability*, i.e., experts using the system should be able to extend the system by adding their expertise into the system. The aim is to increase the efficiency of parameter determination to perform geotechnical finite element calculations and to decrease the variability in input values and results by different engineers. Using a spreadsheet of parameters and equations as input, the system generates a graph showing paths between the parameters and calculates the value for all parameters in the graph. The focus of this study is to determine engineering parameters for coarsegrained soils based on CPT data. However, due to the generality of the proposed method, the system can be extended to a wider range of soils and experimental tests.

Section 2 describes the basic concepts of graph theory and how these are applied in the APD system. Section 3 provides an overview of some of the selected empirical relationships used in the APD system, in order to demonstrate the viability of the system. Section 4 presents a graph generated by the APD system, showing all possible paths between the measured (CPT) parameters and the engineering parameters for a coarse-grained soil, together with the calculated values for all parameters in the graph. Section 5 presents the conclusions of this study.

#### 2 GRAPHS AS DETERMINATION METHOD

#### 2.1 *Brief introduction to graph theory and network analysis*

Graph theory is a branch of mathematics where relationships between objects in a network are studied. A graph is a mathematical term for a network and is described by two sets of objects: *nodes*, representing the entities of the graph, and *edges*, representing the relationship between a pair of nodes. Graphs benefit from their ability to visualise complex systems, like in a *train network* where the connectivity (edges) between the stations (nodes) are modelled and can be used for optimising the transportation between the stations. The parameter determination framework makes use of a *weighted directed graph* in which an inherent direction exists between the pairs of nodes in a graph and edges between the nodes can have a weight. An example of a weighted directed graph is a one-way traffic city centre where the roads (edges) may have a weight representing the distance or travel time.

#### 2.2 *Application to parameter determination in geotechnical engineering*

The concept of graph theory can also be applied to the parameter determination framework, as shown in Figure 1. In this framework, all valid correlation paths are generated, linking measured parameters (e.g., CPT measurements) via intermediate parameters (e.g., relative density) with the constitutive model parameters (Brinkgreve, 2019). It is similar to a satellite navigation system, where commuters can choose between different routes (paths) to travel from location A (source node) to location B (destination node). In a parameter determination system a final parameter (destination node) can be derived in multiple ways (paths) since in many situations more than one empirical correlation may be valid to estimate the same parameter.



Figure 1. Schematic representation of the parameter determination framework; modified from Brinkgreve (2019).

Finding a path in a network from a particular node to another node is a well-known problem, and it is widely used in other applications such as dynamic routing in logistics and computer network routing (Shu-Xi, 2012). Many existing graph algorithms solve the shortest path problem, of which one of the best-known algorithms is Dijkstra's for finding the shortest path between a pair of nodes in a (weighted) graph. There are two main problems when applying existing graph algorithms to the parameter determination framework. First of all, different from graph theory where a path is defined as the connection between one source node (start) and one destination node (end), in the parameter determination framework a path to the destination node can have multiple source nodes since parameters involved in a path can be derived by multivariable formulas (e.g., empirical correlations) that depend on multiple input parameters. In other words, branching paths occur in the parameter determination framework. Therefore, existing graph algorithms are not suitable since they do not deal with branching paths. In fact, according to graph theory a path is defined as the connection between a pair of nodes (i.e., branching paths do not occur). Second, in the parameter determination framework more than one (branching) path can lead to the same destination parameter since often more than one empirical relationship may be available to determine the same parameter. Paths leading to the same parameter may have different lengths, involving different equations that may use different sources or intermediate parameters. This would result in multiple parameter outcomes, which have to be dealt with in the APD system in order to calculate the final parameter value.

To deal with branching paths, the parameter determination framework distinguishes two types of nodes: parameters and methods (

Figure 2). The reason for this is explained by using the graph in

Figure 2a with both parameters and methods as nodes, connected by edges. Parameter C can be estimated in two ways (paths): via Method C1 with Parameter A and B as input, or via Method C2 with Parameter B and D as input. In

Figure 2b, an attempt is made to visualise the same configuration, but now using only parameters as nodes. This representation fails to visualise that there are two unique paths to determine Parameter C; Method C1 and Method C2. Method nodes are often empirical correlations that depend on more than one input parameter (i.e., multivariable formulas). These nodes therefore have multiple incoming edges and indicate that all parameters are required as input for the method. This can be seen in

Figure 2a where both Method C1 and Method C2 have multiple incoming edges illustrating that two branching paths lead to Parameter C. Note that throughout this paper, the term 'method' is used instead of 'formula' or 'correlation' as parameters can also be derived in alternative ways (e.g., tables and charts from literature).



Figure 2. Different representations of a graph with multivariable formulas (methods) using (a) two types of nodes and (b) one type of node.

To keep the APD system modular, the system separates concrete information from abstract information. All the expert knowledge, e.g. empirical correlations, is implemented in an external database of the system (concrete information) while the system itself only contains the algorithm (abstract information) to generate paths between related methods and parameters. Using parameters and methods from the external database as input, the APD system can generate a graph that visualises all valid paths to determine the destination parameters. The system also calculates each parameter in the graph.

It should be emphasised that the responsibility of correctly using the external database lies with the user of the system, i.e., the engineer. However, a standard validated external database of parameters and equations is provided by the APD system. Users who simply use this standard set of parameters and equations should still apply their geotechnical knowledge to confirm or reject the outcome, but even with limited geotechnical knowledge of the user, the system should arrive at reliable outcomes. To establish the external database correctly, the system constructs the paths in reverse order. By beginning at the destination parameter, the system searches in the database what methods can be used to calculate that parameter. Subsequently, for each of these methods, the system 'recognizes' what input parameters are involved in the equation. For these input parameters, the same process is repeated until the path is completed.

#### 2.3 *Generating a path and calculating a parameter*

The key to generating a path is to define the objects in the system in a generic way to link methods and parameters that share a relationship. The relationship between each parameter and method is implicitly defined by the input and output parameters of the method's formula. The system consists of three abstract objects: *Method*, *Connector* and *Parameter*. The methods and parameters from the system's external database are imported into the system, processed by the abstract objects in the system after which the system generates the resulting graph, visualising all valid paths and calculated values for each parameter in the graph. The system is built in the programming language *Python.*

An example graph is shown in Figure 4. Based on the measured cone tip resistance *qc*, pore water pressure *u<sup>2</sup>* and the cone area ratio  $a$ , the corrected cone tip resistance  $q_t$  can be calculated using the empirical relationship proposed by<br>Robertson (1986). To generate this graph, method Robertson (1986). To generate this *Robertson1986* is imported from the external database into the system and becomes a concrete object after implementing the system's abstract object *Method* (Figure 3), which is defined by the external properties: *author, parameters\_in*, *parameter\_out*, *formula*, *accformula* and *weight*. The same applies to the parameters  $q_c$ ,  $u_2$ ,  $a$ ,  $q_t$ , which are imported from the external database into the system and become concrete objects after



Figure 3. External database of the APD system (left), consisting of a parameters and a methods spreadsheet, for the graph example (right)

implementing the system's abstract object *Parameter*, defined by the external properties: *symbol*, *unit*, *definition*, *value*, *accuracy*. These are external properties since they are specified in the external database (see Figure 3 and Figure 4).

The internal properties *connector* and *methods* enable the connectivity between the three abstract objects. These properties are internal since they appear in the APD system, but not in the external database (Figure 3 and Figure 4).



Figure 4. Architecture of the APD system (left) with the abstract objects *Method*, *Connector* and *Parameter* for the graph example (right).

The APD system uses the third abstract object called *Connector* to import each individual method and parameter from the external database into the system, which links methods with parameters based on the output parameter of each method and visualises the graph using the Python graph visualisation library *graphviz* (Gansner, 2011).

The value of a parameter can be calculated based on the information provided by the system's external database (Figure 4). Parameter  $q_t$  does not have a value yet, since it will be calculated by the system. Using the empirical correlation  $q_c+u_2(1-a)$  (*formula*:  $qc+u_2^*(1-a)$ ), proposed by Robertson  $(1986)$  (*author*: Robertson1986), with input parameters  $q_c$ ,  $u_2$  and *a* (*parameters in*: *qc, u2, a*), the output parameter  $q_t$ (*parameter\_out*: *qt*) can be calculated by the system. The same holds for calculating the accuracy using *accformula*.

The term 'parameter accuracy' as used within the framework of this article, can be defined based on the variation in outcomes from the different paths that lead to that parameter, or from variations within the same soil layer. Note that this study does not focus on calculating the final parameter value or the accuracy of a parameter, but it allows these aspects to be included in the system. In fact, the research has meanwhile progressed and a proposal for a final parameter value determination and an accuracy calculation has been presented (Hauth, 2020).

#### 3 SELECTED EMPIRICAL RELATIONSHIPS

#### 3.1 S*tate parameters*

The relative density  $D_r$  is often used as an intermediate parameter, defined as  $D_r = (e_{max} - e)/(e_{max} - e_{min})$  in which  $e_{max}$  is the maximum void ratio,  $e_{min}$  is the minimum void ratio and  $e$ is the in situ void ratio. The selected empirical relationships for estimating the relative density in the APD system are:

- $D_r [%] = (1/2.91)ln(q_c/60(\sigma'_{v0})^{0.7})$  by Lunne & Christofferson (1983), where  $q_c$  is the cone tip resistance and  $\sigma'_{v0}$  is the effective overburden pressure, both expressed in kN/m<sup>2</sup> .
- $D_r [%] = 68[log_{10}(q_{t1}) 1]$  by Jamiolkowski, Ladd, Germaine & Lancellota (1985), where  $q_{t1}$  is the dimensionless normalised cone tip resistance parameter.
- $D_r [%] = 100 \sqrt{q_{t1}} / (305 \cdot OCR^{0.2})$  by Kulhawy & Mayne (1990), where OCR is the overconsolidation ratio.

The overconsolidation ratio OCR is defined as the ratio between the maximum past effective consolidation stress a nd the present effective overburden stress:  $OCR = \sigma_p'/\sigma_{v0}$ . The selected empirical relationship for estimating OCR is:

dilation  $\psi_p$ :  $\phi'_p \approx \phi'_{cy} + \phi_p$ . The following empirical relationships are selected for estimating the peak friction angle  $\phi'_p$  and the dilatancy angle  $\psi_p$ :

- $\phi'_p[°] = \arctan [0.1 + 0.38 \log_{10}(q_t / \sigma'_{v0})]$  by Robertson & Campenalla (1983) and rewritten by Mayne (2006).
- $\phi'_p$  [°] = 17.6 + 11.0 log<sub>10</sub>(q<sub>t1</sub>) by Kulhawy & Mayne (1990).
- $\phi'_p$  [°] = 28 + 12.5 ·  $D_r$ /100 by Brinkgreve, Engin, & Engin (2010).
- $\psi_p$  [°] =  $m[D_r(Q \ln(p') R]$  by Bolton (1986), where  $p'$  is the mean effective stress, R is a fitting coefficient equal to 1, Q is a soil mineralogy and compressibility coefficient ranging from 10 for silica sands to 7 for plane strain conditions and 3 for triaxial conditions Jamiolkowski (2001).
- $\psi_p$  [°] = -2 + 12.5 ·  $D_r$ /100 by Brinkgreve, Engin, & Engin (2010).

#### 3.3 *Stiffness parameters*

Estimating stiffness parameters from in situ tests is difficult since stiffness varies with effective stress levels and stress history, and boundary conditions. The following relationships are used for the



Figure 5. Graph generated by the APD system, visualising the different paths leading from the source parameters (green nodes at the start/top of the graph) to the destination parameters: strength and stiffness parameters for coarse-grained soil (green nodes at the end/bottom of the graph). The calculated value and accuracy for each parameter in the graph are displayed next to the outgoing edge of each method (dashed arrow) for that parameter.

 $OCR$  [-] =  $^{0.33(q_t - \sigma_{v0})^{m'}(p_a/100)^{1-m'}}$ Mayne (1990) with  $m' \approx 0.7$  (Mayne, 2013) and  $p_a$  is by Kulhawy & the atmospheric reference pressure in kN/m<sup>2</sup> .

#### 3.2 *Strength parameters*

The peak friction angle  $\phi_p$  is composed by the ultimate constant volume friction angle  $\phi_{cv}$  and the peak angle of secant stiffness  $E_{50}$ , unloading-reloading stiffness  $E_{ur}$  and oedometric stiffness  $E_{\text{o}ed}$  (all at reference stress level  $p^{\text{ref}}$ ):

- $E_{50}^{ref} = 60,000 \cdot D_r$  by Lengkeek (2003)
- $E_{ur}^{ref} = 180,000 \cdot D_r$  by Brinkgreve et al. (2010)
- $E_{oed}^{ref} = E_{50}^{ref}$  by Schanz & Vermeer (1998)
- $E_{oed}^{ref} = 3 \cdot \sqrt{p_a / \sigma_{v0}'}$  by Vermeer (2000)



Figure 6. External database of methods and parameters used as input by the APD system.

Note that the above empirical relationships (methods) have only been used to demonstrate the functioning of the system. There are facilities in the system that could impose limitations (e.g. regarding soil type, over-consolidation, relative density, plasticity index) on the use of individual methods, to ensure that methods are only used in situations where they are appropriate. A further discussion of this functionality is beyond the scope of this paper.

#### 4 DETERMINING STRENGTH AND STIFFNESS PARAMETERS IN SAND

The concept of graph theory is applied to calculate the Hardening Soil Small-Strain model's strength and stiffness parameters for sand: peak friction angle  $\phi'_p$  (*phiP*), maximum dilatancy angle  $\psi_p$  (*psiP*), reference secant stiffness  $E_{50}^{ref}$  (*E50ref*), reference oedometric stiffness  $E_{\text{pred}}^{ref}$  (*Eoedref*), reference unloadingreloading stiffness  $E_{ur}^{ref}$  (*Eurref*), based on a selection of correlations as input for the system (Chapter 3). The methods and parameters database are defined as comma-separated values (CSV) files. Calculations were verified by comparing the system computed results with hand calculated results, which came out identical.

The graph in Figure 5 involves a hypothetical CPT measurement at an assumed depth of 20 m below ground surface, considering the soil as fully saturated, i.e., phreatic level at the ground surface. The cone tip resistance *q<sup>c</sup>* is assumed 20,000 kPa, the sleeve friction  $f_s$  is assumed 200 kPa and the pore water pressure *u<sup>2</sup>* is assumed 200 kPa. Furthermore, the following standard parameters are used: cone area ratio  $a = 0.8$ , unit weight of water  $\gamma = 9.81 \text{ kN/m}^3$ , the atmospheric pressure  $p_a = 100 \text{ kPa}$ and the rate of stress-dependency of stiffness  $m = 0.7$ . These source parameters are imported into the system from the external

parameters database *parameters.csv* (see Figure 6, lower table). The remaining parameters (i.e., intermediate and destination parameters) do not contain a value and an accuracy since these are calculated using the equations as specified in the methods database *methods.csv* (see Figure 6, upper table).

The graph shows that the APD system is:

- *adaptable*, since the user is able to expand the graph by adding more empirical correlations to the system's external database to calculate the same or new parameters;
- *transparent*, since the graph visualises the entire parameter determination process based on the system's external database which can be modified by the user, giving the user control over the system.

For the purpose of presenting a proof of concept that demonstrates the viability of the APD system, the following assumptions are made in the current system:

- only a limited number of empirical correlations have been used to demonstrate the concept of the system;
- the *weight* of a method was arbitrarily determined, e.g., 1.0 for analytical equations and 0.6 for empirical equations;
- the *accuracy* of a parameter was arbitrarily determined by *accformula* in which the product of the input parameter(s) is multiplied with the weight of the method. The accuracy of the input parameter can be determined from the variation in outcomes from different paths or from variations within a soil layer (Hauth, 2020).

Meanwhile, the research has progressed and has resulted in a prototype that includes 1) reading and interpretation of core CPT data, 2) filtering and stratification into different soil layers, 3) for each layer, parameter determination according to concepts of graph theory (this paper) based on average values per layer, 4) final parameter determination based on averaging of paths outcomes, 5) accuracy calculation based on different paths outcomes and variations in a layer, and 6) creation of parameter sets for a selected soil constitutive model.

#### 5 CONCLUSIONS

This study presents an automated parameter determination system for geotechnical engineering while ensuring transparency and adaptability by applying concepts of graph theory. The system aims to increase the efficiency and consistency of (constitutive) parameter determination for performing finite element calculations and ultimately to decrease the variability results by different engineers. This paper showed how paths can automatically be generated between parameters in a network (graph), using the system's external database as input. The system focused on calculating the engineering parameters for a coarse-grained soil based on CPT data, which was successfully verified by hand calculations.

Adaptability of the system is ensured by separating abstract information, i.e., the algorithm to generate the graph, from concrete information, i.e., the system's external database containing the expert knowledge such as parameters and equations. With only little effort, users of the system, e.g., geotechnical engineers, can simply use this set of parameters and equations to calculate the constitutive model parameters of interest but they may also incorporate their own expertise into the database to confirm or reject the outcome by the system. However, even with limited geotechnical knowledge of the user, the system should arrive at reliable outcomes. Transparency of the system is ensured by visualising the entire parameter determination process in a graph showing all information used by the system as defined in the system's external database which can be verified and controlled by the user.

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